HOS-85-01

Annual Worth Cost Analysis of Pavement Rehabilitation

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Documentation Page

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Author(s): F. W. Jung - Research and Development Branch

Date Published: October 1985

Published by: Research and Development Branch

Contact Person: Pavements & Roadway Office 235-4742

Abstract:

The rehabilitation of pavements is a cycle of gradual depreciation and replacement of the capital invested in the pavement. The capital recovery cost is related to the strength of the pavement and the life-basis for annual worth cost analysis. Whereas the cost of rehabilitation measures comprises a fixed term and a term which varies with overlay thickness, the life-cycle length is dependent on the acceptable minimum performance or serviceability level and the strength of the rehabilitated pavement structure.

These relationships, and a financial equation for annualized costs, are the basic modules of the proposed modelling. The problem of salvage value is avoided by applying the "repeatability assumption" of financial analysis. This means, with regard to pavements, that the same measure of rehabilitation must be used at the same "trigger point" of acceptable minimum performance. Based on earlier modelling of the AASHTO Road Test and Brampton Road Test data, and on inventory data collected and modelled in Ontario, a form for the life-cycle length function has been derived and discussed.

If user costs are not included in the analysis, single overlays done more frequently seem to be economical compared to multi-course overlays and less frequent rehabilitation.

Comments: Prepared for the 1985 Annual Conference of the Roads and Transportation Association of Canada, Vancouver, British Columbia

Key Words:

HOS-85-01

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F.W. Jung
Senior Research Engineer
Research and Development Branch

Published by
The Research and Development Branch
Ontario Ministry of Transportation

For additional copies, contact:
The Editor, Technical Publications
Room 320, Central building
1201 Wilson Avenue
Downsview, Ontario
Canada M3M 1J8

Telephone: (416) 235-3480
Fax: (416) 235-4872

October 1985

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1/ INTRODUCTION

1.1/ Capital Investment and Depreciation

The rehabilitation of pavements can be understood as part of the financial management of highway investments. Each section of a road periodically deteriorates and must subsequently be restored to its full level of service. What is being restored is a certain amount of depreciated capital. Depreciations can occur through obsolescence when roads are re-routed or become redundant due to a loss of traffic, etc. Depreciation in the context of pavement rehabilitation, however, is mainly physical, i.e. a decrease in performance or serviceability because of the deterioration of surface layers and/or support structures.

This concept of capital depreciation of pavement investment is illustrated in Figure 1. It is shown that total cash flows for rehabilitation can be at different levels -- sufficiently high (X) to maintain the value of investment, or lower (Y and Z), resulting in a smaller or larger loss of value (A_2 and A_3) during a certain period of time (N). The diagram in Figure 1 is valid with regard to all levels of planning, from local to regional or national road networks.

The actual life-cycle costs (X, Y, or Z) for a planning horizon, N, depend on the pavement performance as a function of time. The use of performance indicator curves in life-cycle cost analysis has been proposed and discussed before [1], most recently in [2] and [3]. It is necessary to look at this in more detail, with respect to the concepts of financial analysis.

Periodic capital investment and annual depreciation in correspondence to performance curves is illustrated in Figure 2. In this figure, capital recovery costs (C.R.) are calculated for two cases, A and B, using compounding factors as defined in Reference [7]. Complete depreciation of the rehabilitation capital spending at year zero, say, 110 000.00 is reached at PCI = 40, after 12 years. Early rehabilitation after 10 years (A) results in a salvage value of, say, 25 000.00 and postponed rehabilitation would lead to a negative salvage value (or additional cost) of 35 000.00 (B). The actual figures are not significant as they are used only to illustrate and clarify the concepts of depreciation and salvage value of a rehabilitation expenditure.

The modelling of life-cycle costs, as presented here, deals with one pavement section of known performance characteristics. However, it is generally true for any section that pavements, and other major parts of the transportation infrastructure, after being constructed, go through cycles of gradual deterioration and periodic rehabilitation. Any measure of rehabilitation, such as a bituminous overlay, can be regarded as a capital expenditure to replace a depreciated asset. The "when and how to replace" decision is similar to that of a firm when it must decide on the replacement of machines in its shop, or of trucks in its fleet. A life-cycle cost analysis can help to find the most economical replacement schedule.

2/ PERFORMANCE LEVEL AND LIFE-CYCLE

As long as pavement performance is maintained within acceptable limits, there is no need to consider the benefit side of the economic equation because any advantage gained through performance differences is uncertain and probably very small. Thus, the economic study may be confined to minimizing the cost. However, it is important to annualize the cost over the true life-span or cycle, and in such studies it is customary to assume that expenditures and performance cycles are repeated. As in the case of any capital asset, this means that the depreciated part, after its life-span, has to be replaced by a new part of equal first value. In this way, present worth or equivalent annual worth costs can be calculated, which depend on the life-span and its corresponding or related degree of deterioration in terms of performance (the "PCI-drop").

As future decisions on the timing and the particular measure of rehabilitation are uncertain, life-cycle costs should be based on the same PCI-drop, and on the same kind of rehabilitation measure after each replacement period. This corresponds to the well known "repeatability assumption" in financial analysis.

The relationship between first cost, C, and the ensuing performance cycle is illustrated in Figure 3, which also shows how the cycle is assumed to repeat. Only the first shaded terms are used in the analysis. Note that the life-cycle, N, in conjunction with the performance trigger point, Y_L, is the most important technical parameter of this cost study, rather than the exact shape of the performance curve. If an analytical expression of the performance curve is given, then the value N can be calculated [4].

Thus, in assuming consistent periodical performance drops and jumps, there is a "first level" optimization which is related to the choice of PCI-jump or magnitude of allowable deterioration and its related rehabilitation cost, C. Note that this choice of deterioration jump and rehabilitation measure entails a choice of life-span or life-cycle length, N. Also note that salvage values, as illustrated in Figure 2, do not play a part in this approach to modelling.

The methods of pavement rehabilitation range from relatively inexpensive treatments, such as hot-mix patching or single course overlays, to heavy expensive treatments, such as multi-course overlays with crack sealing and padding. In general (i.e., all other circumstances being equal), the heavier treatments last longer. This fact entails a "second level" of optimization which is related to the choice of stronger and weaker designs as illustrated in Figure 4. This choice again influences the life-cycle length.

3/ PAVEMENT STRENGTH AND REHABILITATION COST

One of the most important technical facts to be considered is that the life-span, N, is a function of the added pavement strength after rehabilitation. Depending on the strength of the overlay, the life-span or life-cycle length, N, can be shorter or longer than the length of the previous cycle, as is illustrated in Figure 4. Again, the actual shape of the performance curve is not important, except that it determines the trigger point, Y_L, and the cycle length, N. Further, even with this fact in mind, the repeatability assumption is applied to the future, assuming the same treatment is repeated with the same ensuing life-span as the current one, so that the concepts shown in Figure 3 remain valid.

In studying data on various rehabilitation costs, C, it is certain that such costs are also a function of pavement strength. At this point, it is advantageous to remember the concept of equivalent thickness [5]. Depending on the material, various courses of overlay have thicknesses which could be converted into an equivalent surface course thickness, t, by an equivalency factor, e. This idea requires further study.

Cost data of 1984 have been processed in this way and are plotted in Figure 5. In this diagram the solid lines represent the cost as a function of bituminous layer thickness. The dashed line is an attempt to establish such a function based on equivalent surface layer thickness. In this case the granular bases can be included with an assumed equivalency factor of e = 1/3.

The general form of a cost-thickness relationship is probably the usual combination of fixed and variable cost, as shown in Figure 5 (broken line). Alternatively, the function could be processed as a table of benchmark costs for different measures related to discrete overlay thicknesses and other features. In general, it should be relatively easy and straightforward to determine the functional relationship between costs and overlay thicknesses for bituminous layers, although such cost functions are subject to change with time and regional conditions. However, it is more difficult to estimate the life-cycle length, N, as a function of thickness, t, even if the PCI-jump is fixed.

The OPAC performance prediction model, developed in 1974/76 [6], can be used to calculate the life-cycle length, N, as a function of overlay thickness, t, and the minimum acceptable level of performance, Y_L. In spite of the fact that this prediction model is limited, one must recognize that such a model is needed to determine the relationship between the strength, t, of an overlay and the ensuing life-span, N, for a chosen lower limit of performance, Y_L. Figure 6 shows the result of a calculation for two different designs of a secondary road, with a silt (1) and a soft clay (2) as subgrade. The lower limit of performance has been chosen alternatively as $Y_L = 45$ and as $Y_L = 55$. As expected, the life-cycles are longer for $Y_L = 45$, the lower of the two limits. In the relevant range between 40 mm and 150 mm overlay thickness, the curves can be expressed approximately by a function of the form:

$$N = A\ t^b$$

in which A and b are constants dependent on the chosen Y_L value and other conditions.

4/ ECONOMIC MODEL

The relationships described so far, and shown in Figures 3 to 6, can be expressed as a simple economic model by the following set of equations, which are all to be understood as simple prototypes of various parts or modules of the model:

$$C = C_c + C_v t \quad (1)$$

$$N = A\ t^b\ ;\quad < 25 \text{ years} \quad (2)$$

$$Q = C \cdot i \cdot (1+i)^N / ((1+i)^N - 1) + M \quad (3)$$

where: C_c = constant rehabilitation cost, per km (2 lanes).
C_v = variable rehabilitation cost, per km, per mm.
t = equivalent surface layer thickness of rehabilitation overlays in mm.
C = rehabilitation cost, first cost, per km (2 lanes).
M = annual maintenance cost, per km (2 lanes).
N = life-cycle period, in years.
i = combined interest rate, in % ÷ 100 (real interest rate and rate of inflation).
Q = annualized life-cycle cost or annual worth cost, per km, per 2 lanes.
A, b = constants for function $N = A\ t^b$

NOTE: The "fixed" or "constant" cost, C_c, is only constant with respect to the thickness, t, as an independent variable. In reality, C_c and, to a lesser extent, C_v are functions of other conditions such as contract size (number of kilometres), and remoteness of location.

Neglecting the annual maintenance cost, M, and assuming an interest rate of zero, the annualized life-cycle cost is simply $Q = C/N$. For the variable part of C only, the following simple relationship can be derived:

$$Q_v = C_v \cdot t^{(1-b)}/A \tag{4}$$

For this case, according to Equation 4, the annual life-cycle cost increases with t, for $b < 1$; and it decreases with t, for $b > 1$; whereas it is indifferent for $b = 1$. When an interest rate is introduced, using Equation 3, the life-cycle cost increases even more with thickness, t, as higher interest rates favour solutions of lower first cost, C. The conclusion is that expensive multi-course overlays are economical only if the parameter b is distinctly larger than one.

A method to calculate life-cycle periods, N, is presented in Appendix A, based on Reference 6, which is the performance prediction part of the OPAC model (Ontario Pavements Analysis of Costs).

The calculations based on the OPAC model are summarized and plotted in Figure 6. The curves show that b is most probably smaller than one, at least for all the curves shown in the figure. This would mean that inexpensive single course overlays combined with shorter ensuing life-cycles are more economical, but only if user costs are small enough to be neglected.

Equation 1 represents the relationship between the first costs of rehabilitation measures and the strength or thickness of overlays. Equation 2 is a prototype for the relationship between the strength or thickness of overlays and the life-cycle length, which is significant as it is also a function of the chosen lower limit of performance (Y_L). Equation 3 is the known formula of financial analysis.

With reference to Figure 6 and Equation 2, the optimization on the project level can be understood as follows:

Level 1: Depending on the choice of the lowest allowable limit of performance level (Y_L or PCI_L), there are shorter or longer life-cycles, N, which influence the compounding factor (Equation 3) and the present or annual worth cost.

Level 2: Depending on the shape of the curves in Figure 6 (being concave when looking from below), the exponent, b, is smaller than one. Therefore, the annual worth cost increases with increasing thickness, t, at least for the cases presented here and calculated by the OPAC method. This leads to an optimization along the constraint boundary of minimum overlay thickness.

Example

With reference to the examples presented in Figure 6, costs are calculated for a subgrade modulus of M_S = 2700, the 762 mm subbase on clay subgrade. Optimization is achieved by minimizing the annual worth costs, Q, based on Equations 1 to 3. The constraints can be visualized as the minimum acceptable level of performance and as the smallest possible thickness of overlay:

1. min Y_L = 45 (upper curves in Figure 6)
2. min t = 40 mm (1.5")

The combined interest rate is assumed to be 11%. The following equations apply:

$$C = (10 + 0.50\ t) \times 1000 \quad \text{(refer to Figure 5)}$$

$$\left.\begin{array}{l} N = 3.65\ t^{0.357};\ \text{for } Y_L = 45 = \min Y_L \\ N = 3.94\ t^{0.265};\ \text{for } Y_L = 55 = \min Y_L \end{array}\right\} \text{Curve-fitted from Fig. 6.}$$

$$Q = C \times 0.11 \times 1.11^N/(1.11^N-1); \quad \text{for } M = 0$$

Annual worth costs, Q, have been calculated for various overlay thicknesses, t, and the results are presented in Table 1. The following observations are made:

For this secondary road of lower traffic volume, a rehabilitation strategy of low minimum performance limit (Y_L = 45) seems to be economical, gaining in life-cycle length without too much additional deterioration. However, there is no gain in economy by going beyond a minimum layer thickness (40 or 50 mm) when applying an overlay at the chosen trigger point of Y_L = 45. However, the economy of a stronger overlay may improve when user costs are taken into account, because the optimum solution of t = 40 mm entails more frequent construction work (lane closures) and rougher driving conditions [2].

Table 1, Example

t (mm)	$Y_L = 45 = \min Y_L$		$Y_L = 55 > \min Y_L$	
	Q [Can. $]	N [Years]	Q [Can. $]	N [Years]
40	4350.00	13.6	4964.00	10.5
50	4901.00	14.8	5609.00	11.1
75	6286.00	17.0	7207.00	12.4
100	7668.00	18.9	8780.00	13.4

5/ COMPARISON WITH PERFORMANCE PREDICTION IN PARS

It has been demonstrated that the life-cycle, N, can be expressed as a function of overlay thickness (or strength) by Equation 2, namely:

$$N = A\ t^{b}$$

where A and b are dependent on the performance trigger point, Y_L, and where b is less than one. This was based on a model derived from the AASHTO and Brampton Road Test data [6]. Later, in the development of the PARS model [8], pavement performance was modelled differently, using inventory data collected in Ontario during or before 1978. The same form of function as expressed by Equation 2 can be derived from the PARS modelling. In accordance with an unpublished report, the PARS performance modelling has the following formula:

$$Y = 95 - K\ X^{a}\ t^{-b'}\ T^{C} \qquad (5)$$

where: Y = Performance Condition Rating (PCR or PCI)
K = Coefficient
X = Time in years after rehabilitation
t = Thickness of overlay in mm
95 = Maximum Y
T = traffic in terms of AADT, (Annual Average Daily Traffic)
a,b',c = constants

The coefficients and constants depend on the class or group of roads which are identified as exhibiting similar performance.

For a certain chosen trigger point, $Y = Y_L$, as the lowest acceptable performance index, the life-cycle, X = N, can be calculated by the following equation, derived from Equation 5.

$$N = \left[\frac{95 - Y_L}{K\ T^{c}}\right]^{\frac{1}{a}} \times t^{\frac{b'}{a}} \qquad (6)$$

Equation 6 has the same form as Equation 2. Note that, with regard to the coefficients:

$$A = \left[\frac{95 - Y_L}{K\ T^c}\right]^{\frac{1}{a}} \qquad (7)$$

$$b = \frac{b'}{a} \qquad (8)$$

Example:

The following coefficients were derived for the Southwest and Central Regions of Ontario, by regression analysis of 1978 data, for "average" performance:

K = 4.8306   a = 1.0894
C = 0.2202   b' = 0.6358

Assuming an AADT of T = 2000, the following values can be calculated for trigger points of Y_L = 40, 50, and 60:

a) $$b = \frac{b'}{a} = \frac{0.6358}{1.0894} = 0.5836$$

b) $$A_{40} = \left[\frac{55}{4.8306 \times 2000^{.2202}}\right]^{0.91793} = 2.006$$

$$A_{50} = \left[\frac{45}{4.8306 \times 2000^{.2202}}\right]^{0.91793} = 1.669$$

$$A_{60} = \left[\frac{35}{4.8306 \times 2000^{.2202}}\right]^{0.91793} = 1.325$$

Equation 2 can now be plotted for the above coefficients, to illustrate the trend of the life-cycle function with regard to the variable t and

Y_L, refer to Figure 7. Life-cycle lengths according to this PARS model are longer, especially for multiple overlays.

The life-span function $N = A\ t^b$, with $b < 1$, and A and b depending on the performance trigger point Y_L, traffic, etc., seems to be well established in its basic form or trend. Because of the limitations of the underlying data (OPAC, PARS), it is only valid for overlays between approximately 40 mm (1.5 in.) and 150 mm (6 in.). Below and above these limits, the life-spans could be much shorter but not much longer than calculated. The OPAC model, with generally lower values of N and with smaller exponents, b, seems to be closer to reality, although this is still subject to further analysis of data. There are two kinds of analysis that should be further explored:

1) Inventory data on performance should be processed by grouping road sections into more homogeneous classes with identifiable strength and traffic characteristics, so that N and b can be determined with more certainty (less variance).

2) Experimental data may improve structural performance prediction modelling beyond the present limitation of OPAC [6].

6/ CONCLUSIONS

An economic model for the life-cycle or annual worth cost analysis of pavement rehabilitation of a particular road section (project level) as outlined previously is based on several equations or functions, namely:

1) First cost as a function of overlay strength or thickness, Equation 1.

2) Life-cycle length as a function of pavement performance standard and overlay strength or thickness, Equation 2.

3) Annual worth cost, Equation 3.

Optimization based on this model has been illustrated by minimizing life-cycle costs, with constraints on pavement performance level and overlay thickness, by specifying minimum values.

Whereas a lowering of the performance trigger point and the ensuing increase in life-cycle length appears to be economical, it seems to be uneconomical to increase the overlay thickness beyond a minimum required design value. This is only true if user costs are negligible.

The model of annual worth cost analysis outlined in this paper depends on valid information on costs for various rehabilitation measures, preferably expressed as a function of overlay thickness. More difficult to obtain is valid information on the life-span or life-cycle length as a function of overlay strength or thickness, for selected lowest acceptable PCI-trigger points of rehabilitation. This requires a comprehensive performance prediction model similar to the one presented by the author in 1975 [6]. A computer program of this "OPAC" prediction model was used to illustrate the function $N = A \times t^b$. The examples chosen were pavements of low strength, for which the model predicts better than for stronger pavements.

Finally, the performance modelling of PARS [8] was used to determine the coefficients A and b in Equation 2 (based on 1978 data), for average performance in the Regions of Ontario.

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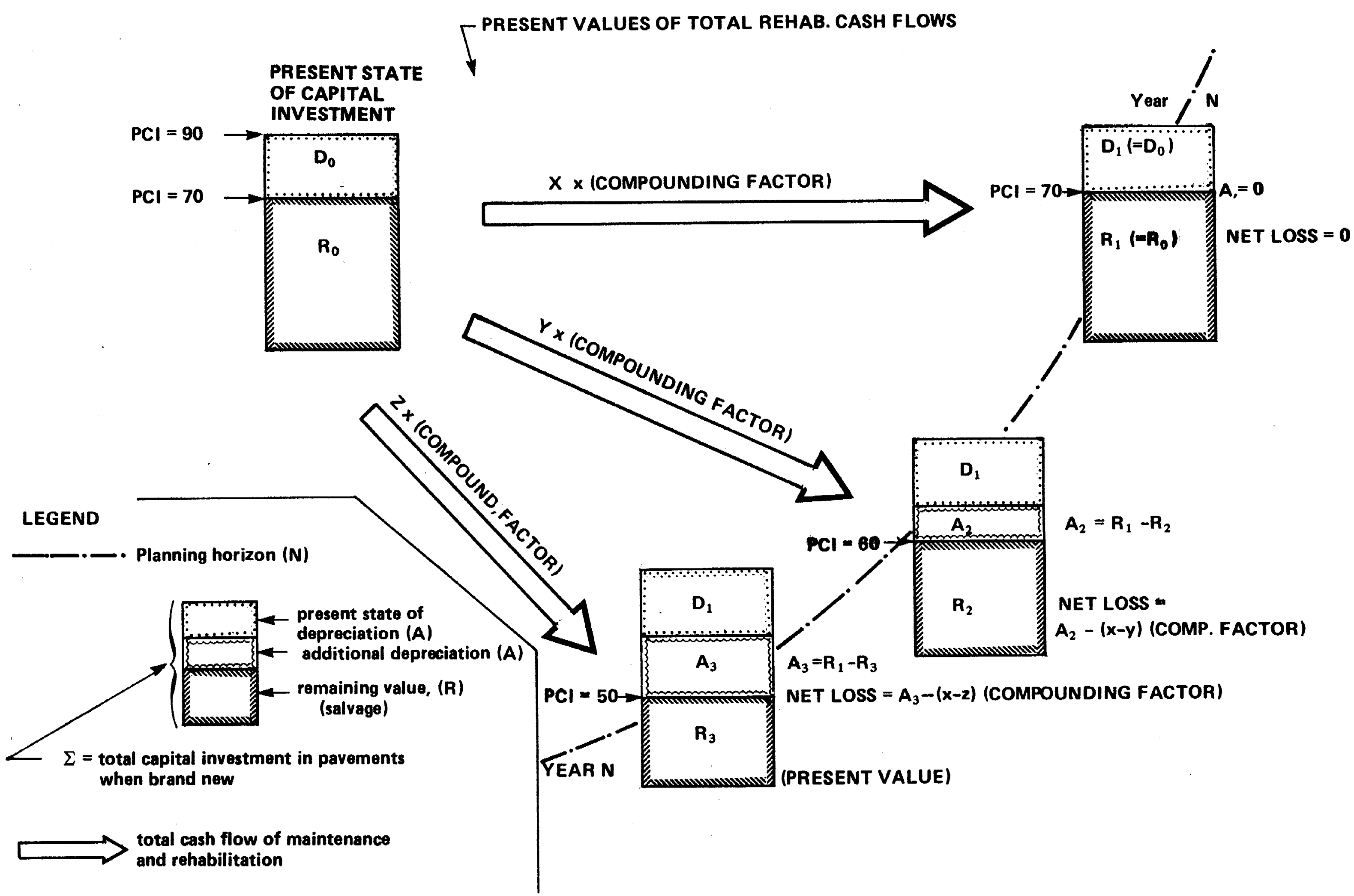


Figure 1/ Capital Depreciation Scenarios of Pavements

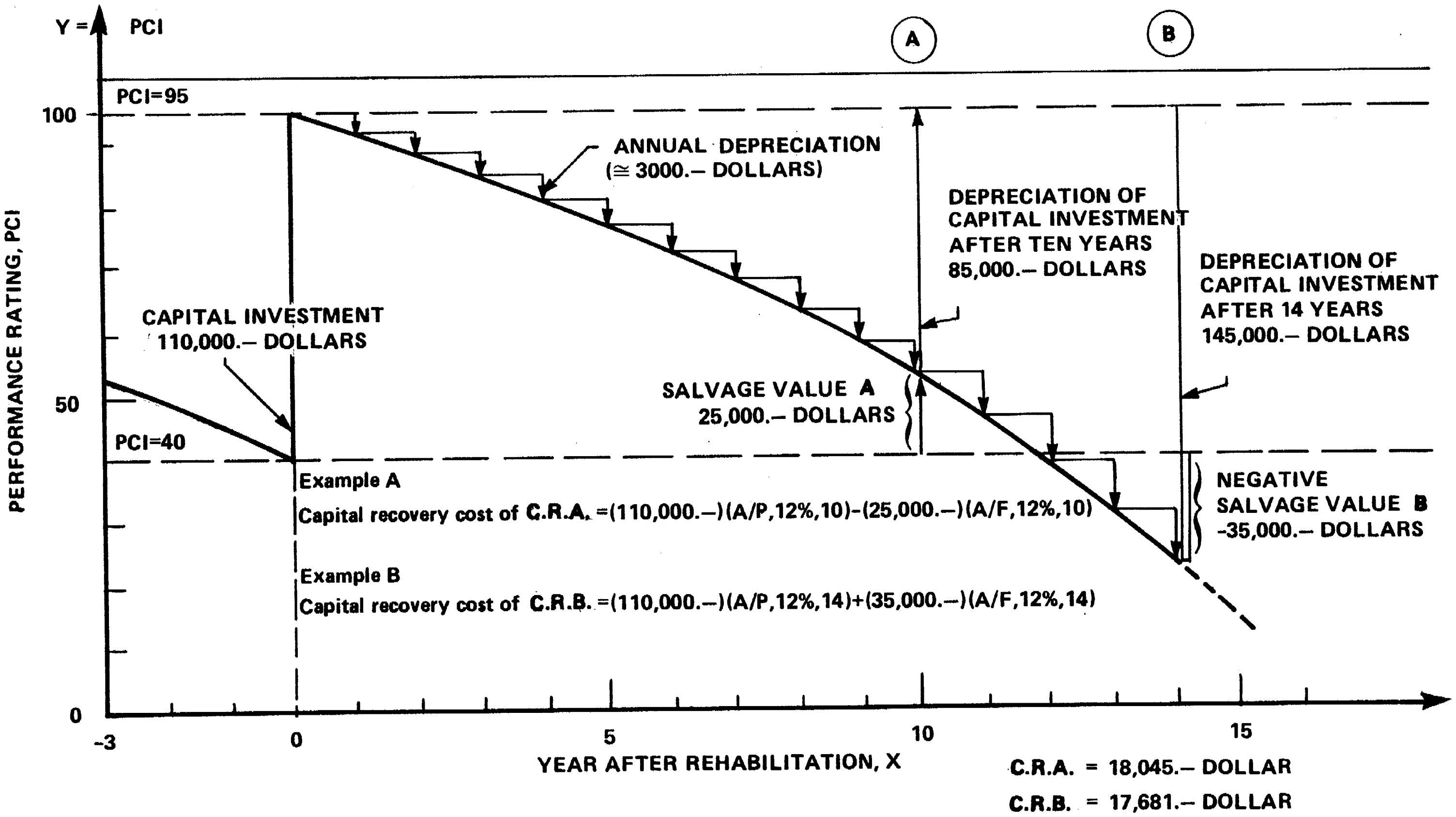


Figure 2/ Complex Example of Investment Depreciation and Capital Recovery Costs

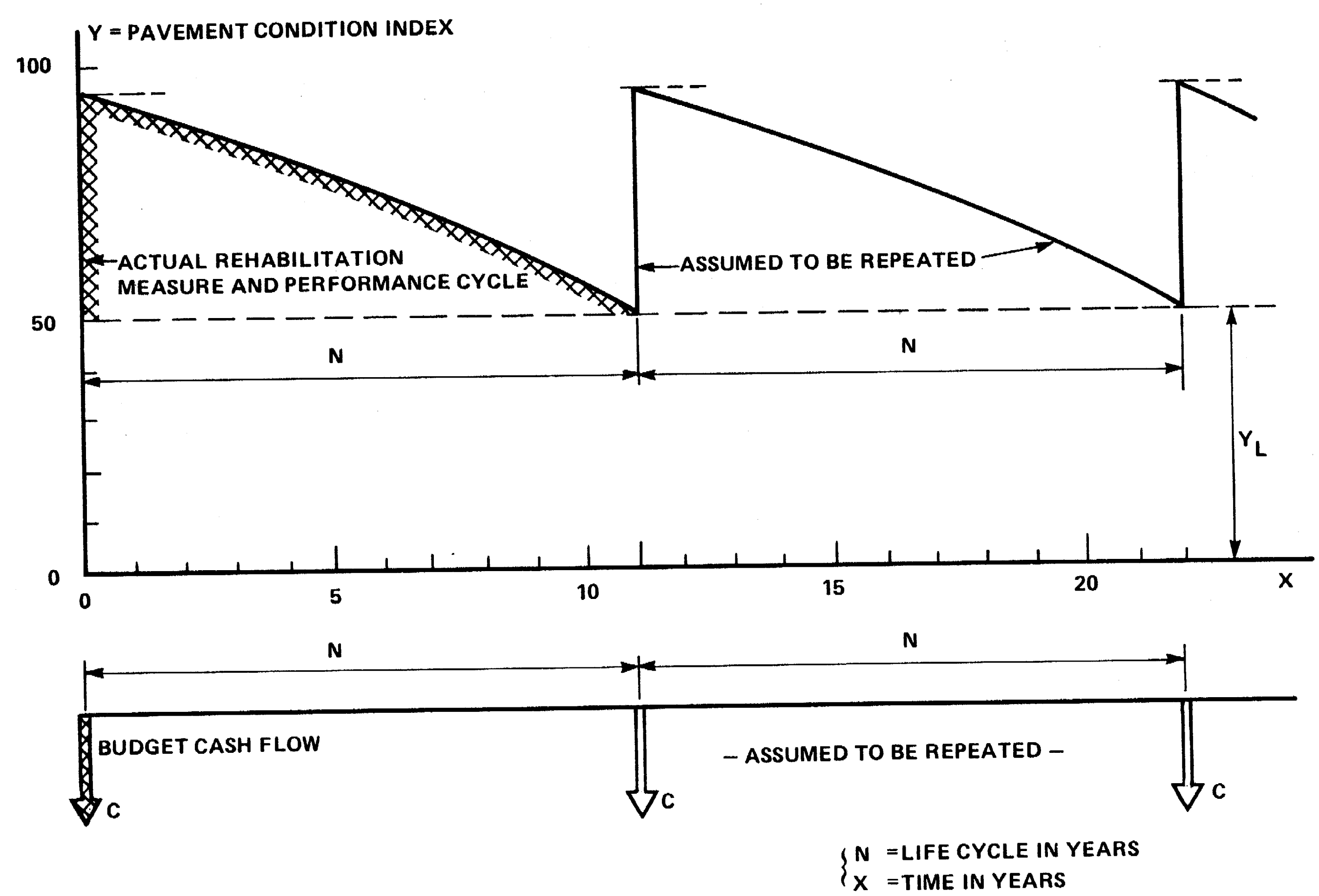
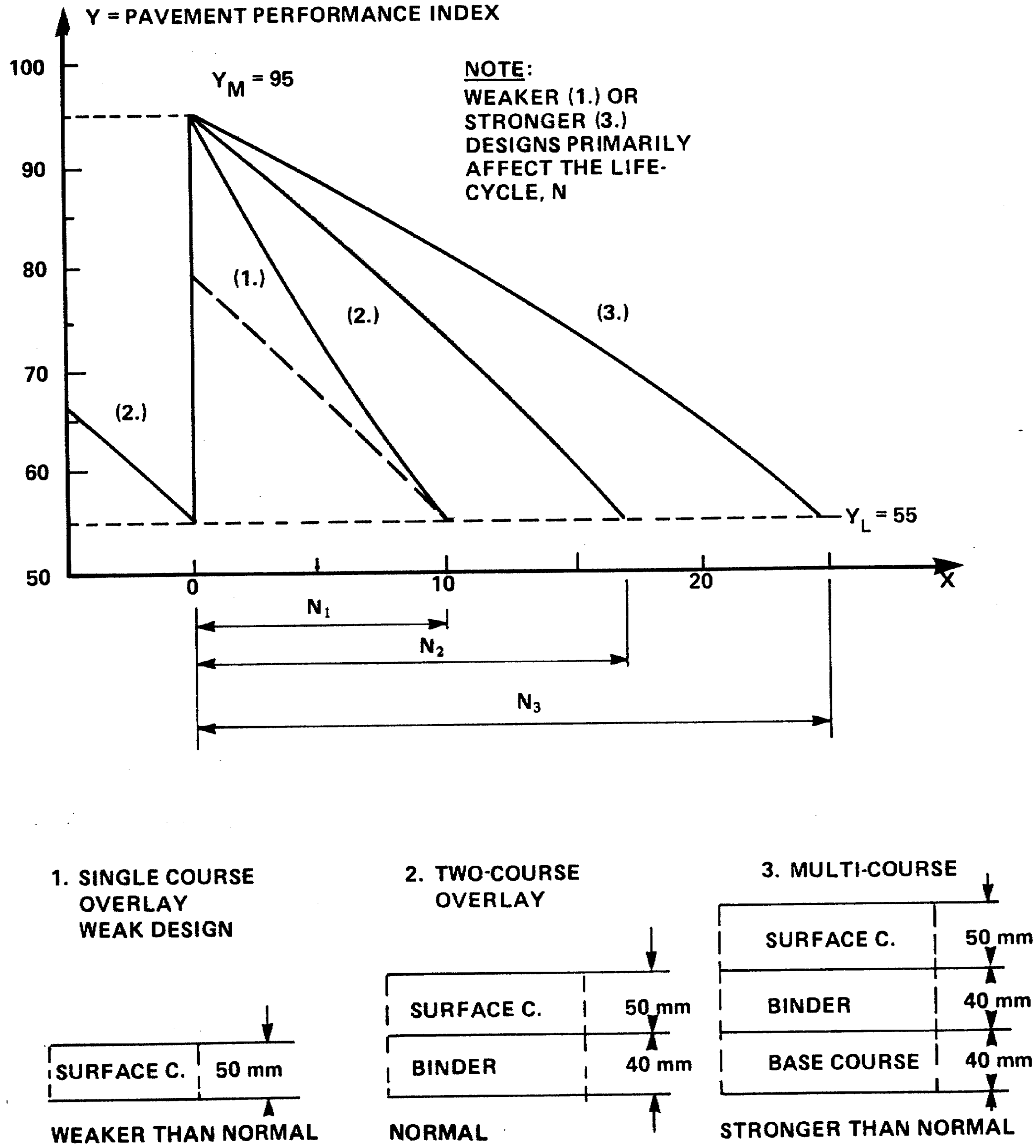


Figure 3/ Repeatability Assumption for Matching Cash Flow and Performance Cycle



NOTE: "NORMAL" MEANS A DESIGN FOR THE SAME LIFE-CYCLE AS BEFORE (2.)

Figure 4/ Alternative Rehabilitation Treatments and their Effect on Performance

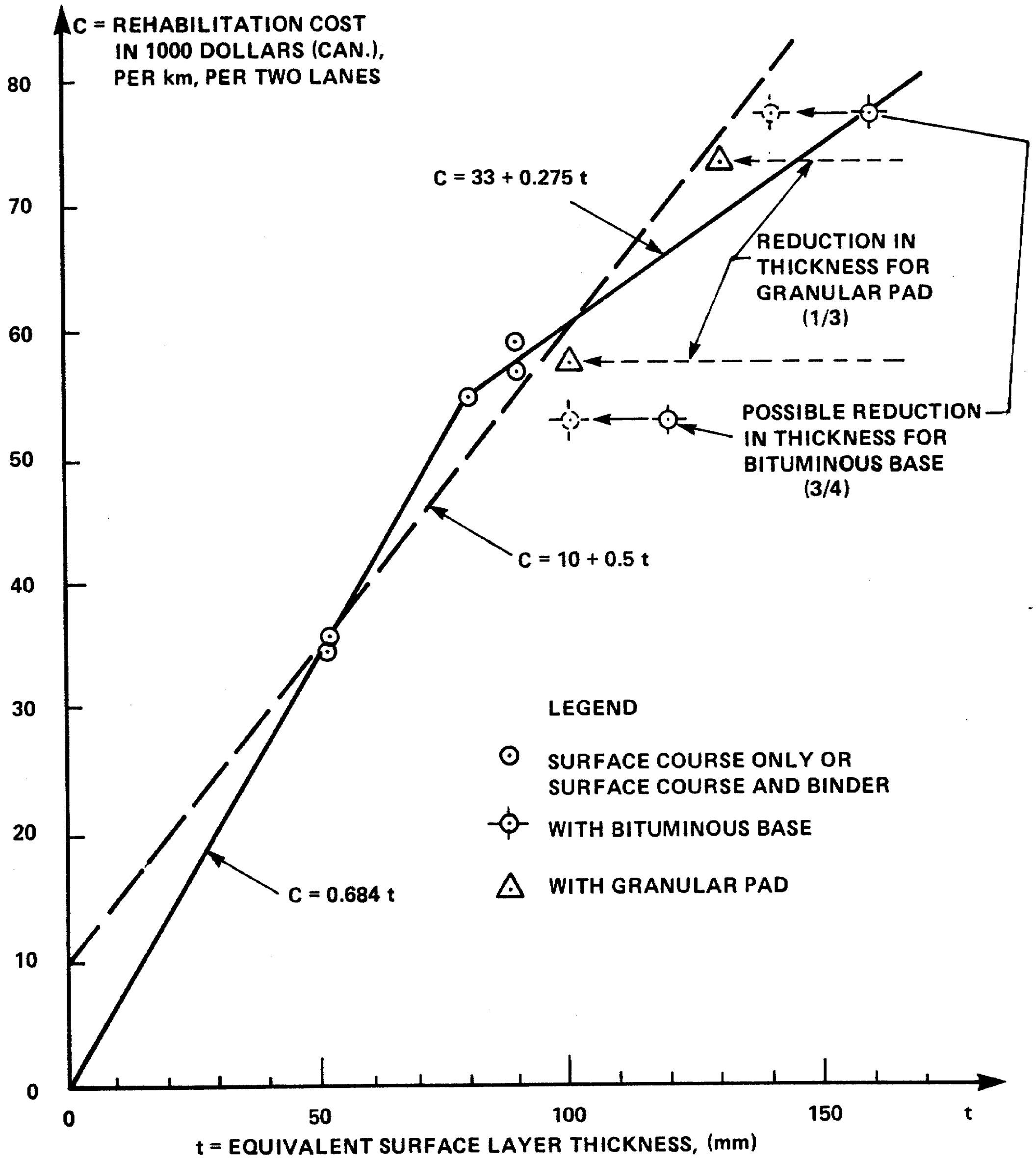


Figure 5/ Cost Functions (from 1984 cost data)

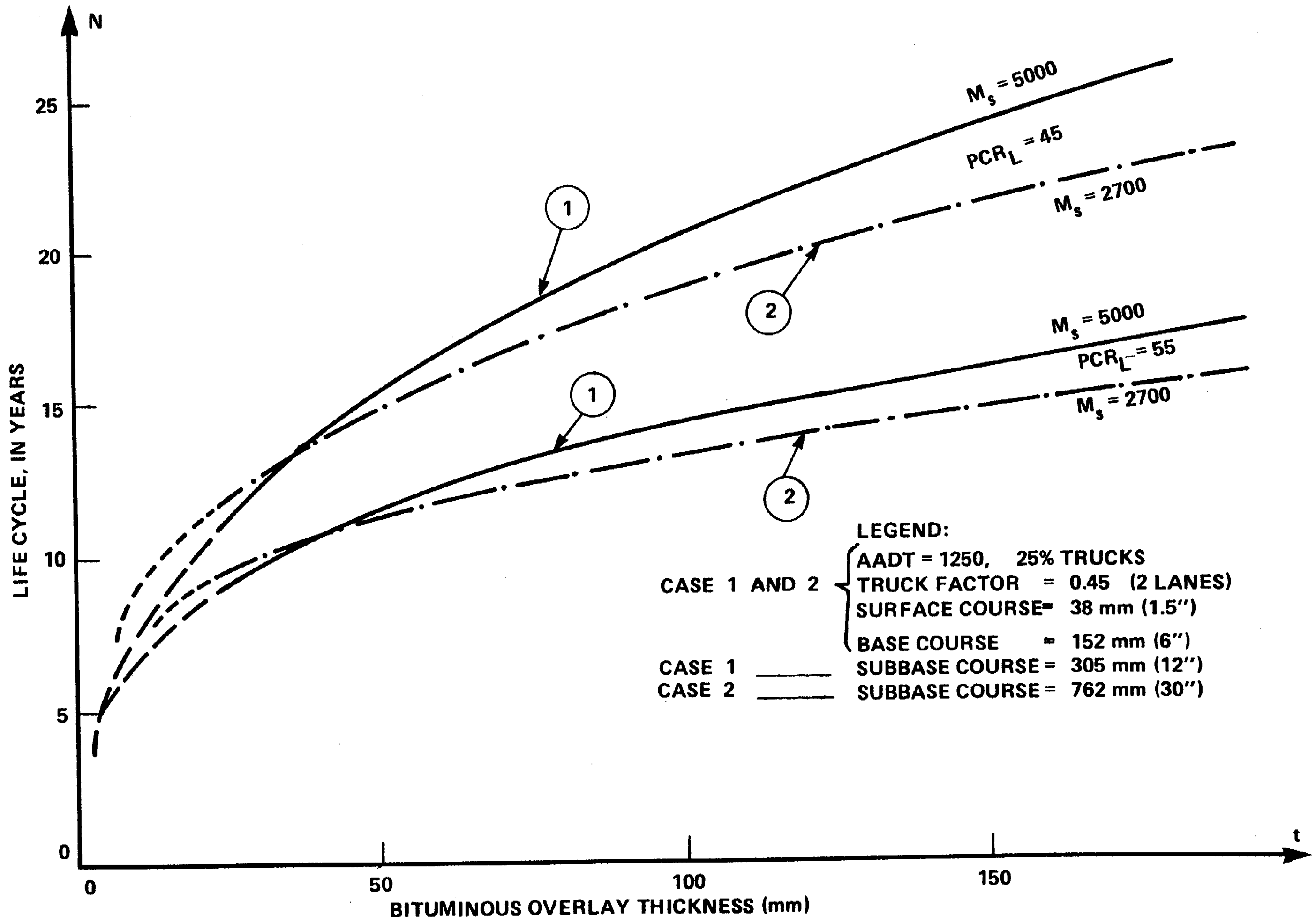


Figure 6/ Life Cycle Length versus Overlay Strength (Thickness)

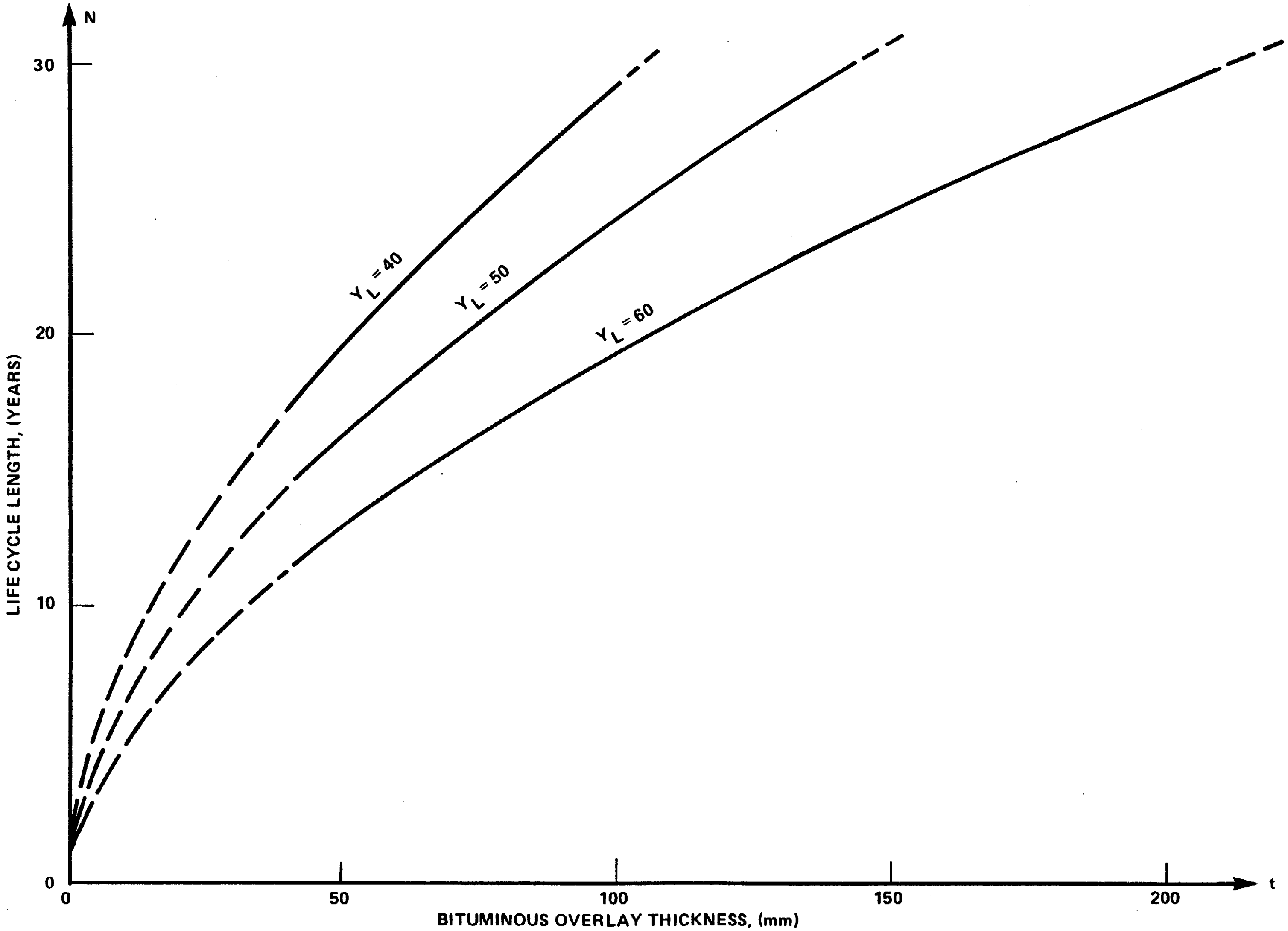


Figure 7/ Life Cycle versus Overlay Strength (Thickness), PARS

APPENDIX A, COMPUTER PROGRAM FOR PERFORMANCE PREDICTION OF ASPHALT PAVEMENTS

The computer program presented in this appendix is based on the K.B. Woods Award paper, 1975, presented to TRB: "A Performance Prediction Subsystem", by Friedrich W. Jung, Dr.R. Kher, and W.A. Phang, MTC/R&D Research Report RR 200, August 1975. Except for including a later modification for small Odemark Deflections (W = 0.025 inches), the program follows the model as given in the paper.

At present the Minisry (MTC Ontario) uses a different measure of performance, the Performance Condition Rating or Index, PCR or PCI. To account for this, the Rriding Comfort Index of the original model (from 1 to 10) has been scaled up by a factor of 95/75 (except for the residual strength of old asphaltic layers after overlay which was scaled up by a factor of 100/10). Equivalency factors for overlay courses, based on surface layer strength, have been introduced as a projected model extension.

Notations:

AI = Initial Average Annual Daily Traffic (AADT(i))
AF = Final Average Annual Daily Traffic (AADT(f))
DS = Number of days per year of truck traffic (DAYS)
AP = Analysis period in years (Ap)
PO = Riding Comfort Index at time = 0
T = Initial truck percentage (%/100)
TF = Final truck percentage (%/100)
LI = Initial lane distribution factor
LF = Final lane distribution factor
FI = Initial Truck Factor (TF(i))
FF = Final Truck Factor (TF(f))
W = Odemark Deflection after overlay
W1 = Odemark Deflection of original new pevement
Y = Time in years
P = Pavement Riding Comfort Index
MS = Subgrade layer coefficient (in psi)

A1 = Equivalency Factor for bituminous surface courses
A2 = Equivalency Factor for Granular A base course
A3 = Equivalency Factor for subbase
HO = Bituminous overlay thickness, in mm
H1 = Bituminous layer thickness before overlay, in mm
H2 = Thickness of granular base course before overlay, mm
H3 = Thickness of subbase layer before overlay, in mm
T1, T2, T3 = Thickness of various overlay courses, in mm
E1, E2, E3 = Equivalency Factors for overlay strength
RC = Pevement Condition Rating or Index
RL = Lower Limit of PCR or PCI

The meaning of other notations found in the BASIC program can be easily assertained by comparing with the model in the aforementioned paper.

APPENDIX A   BASIC Program Print-Out

```
10 REM  PERFORMANCE PREDICTION
20 REM  FILE NAME: PP-N/O
30 READ RL
40 READ MS, AP, PO
50 READ AI, AF, DS
60 READ T , LI, FI
70 READ TF, LF, FF
80 READ A1, A2, A3
90 READ H1, H2, H3
100 READ E1, E2, E3
110 READ T1, T2, T3
120 LET HE = (A1*H1+A2*H2+A3*H3)/25.4
130 LET Z1 = 0.9*HE*(50000/MS)↑(1/3)
140 D1 = 2*MS*Z1*SQR(1+(6.4/Z1)↑2)
150 W1 = 9000/D1  :  BB = 60
152 INPUT "NEW PAVEMENT OR OVERLAY (N/O)";A$
154 IF A$ = "O" GOTO 160
156 LET W = W1 : GOTO 230
160 LET K = 0.48 + 8*W1
170 LET K1 = 0.3 + 0.68*(RL/100)
180 LET HO = E1*T1 + E2*T2 + E3*T3
190 HF = (A1*HO+A1*K1*H1+A2*K*H2+A3*K*H3)/25.4
200 Z = 0.9*HF*(50000/MS)↑(1/3)
210 D = 2*MS*Z*SQR(1+(6.4/Z)↑2)
220 LET W = 9000/D
230 OPEN 4,4,2
240 PRINT : PRINT#4
250 PRINT " W1=";W1;"  W =";W;" IN"
255 PRINT#4," W1=";W1;"  W =";W;" IN"
260 PRINT " W =";W*25.4;" MM"
265 PRINT#4," W =";W*25.4;" MM"
270 NF = (AP*DS/4)*(AI*T*LI*FI+AF*TF*LF*FF)
275 PRINT#4 : PRINT#4 : PRINT
280 FOR Y = 1 TO AP  STEP 1
290 N = (NF/AP)*(2*AI*Y/(AI+AF) + Y*Y*(AF-AI)/(AP*(AI+AF)))
295 IF W < 0.025  THEN GOTO 310
300 PS = 1000*N*W*W*W*W*W*W : GOTO 320
310 PS = N/(1000000*(28.672-983.04*W))
320 PT = 2.4455*PS + 8.805*PS*PS*PS
330 PE = (PO - PO/(1+BB*W))*(1 - EXP(-.06*Y))
340 LET P = PO - PT - PE
350 RC = INT(100*P*95/PO + 0.5)/100
360 N = INT(N+0.5)
370 PRINT Y, N, RC
380 PRINT#4, Y, N, RC
385 IF RC < RL GOTO 392
390 NEXT Y
392 PRINT : PRINT#4
394 PRINT"LIFE SPAN =";Y-1;" YEARS"
396 PRINT#4,"LIFE SPAN =";Y-1;" YEARS"
400 CLOSE 4
```

APPENDIX A, Complete Data Set and Test Example

Data

```
500 REM  DATA SET
510 DATA 45 : REM  LOWER LIMIT OF PCR
520 REM MS,     AP,   PO
530 DATA  3500,  25,  7.5
540 REM   AI,    AF,    DS
550 DATA 2000, 10000, 300
560 REM   T ,    LI,    FI
570 DATA 0.050, 0.80, 4.0
580 REM   TF,    LF,    FF
590 DATA 0.060, 0.80, 6.0
600 REM   A1, A2, A3
610 DATA  2,  1,  0.66666667
620 REM   H1,  H2,  H3
630 DATA  100, 150, 200
640 REM   E1,  E2,  E3
650 DATA  1,   0.8, 0.8
660 REM   T1,  T2,  T3
670 DATA  50,  40,  40
680 END
```

Test Example

```
W1= .030579545   W = .0267286928  IN
W = .678908796  MM
```

1	86400	90.61
2	185600	86.25
3	297600	81.86
4	422400	77.34
5	560000	72.56
6	710400	67.35
7	873600	61.46
8	1049600	54.59
9	1238400	46.33
10	1440000	36.19

LIFE SPAN = 9 YEARS

APPENDIX A, Program to Calculate Life Span for Lower Limit of PCR

```
20 REM  FILE NAME: PP-LL1
25 DIM RC(30) :  BB = 100
30 READ RL
40 READ MS, AP, PO
50 READ AI, AF, DS
60 READ T , LI, FI
70 READ TF, LF, FF
80 READ A1, A2, A3
90 READ H1, H2, H3
100 OPEN 4,4,2
120 LET HE = (A1*H1+A2*H2+A3*H3)/25.4
130 LET Z1 = 0.9*HE*(50000/MS)↑(1/3)
140 D1 = 2*MS*Z1*SQR(1+(6.4/Z1)↑2)
150 W1 = 9000/D1
160 LET K = 0.48 + 8*W1
170 LET K1 = 0.3 + 0.68*(RL/100)
180 FOR HO = 50 TO 130  STEP 32
190 HF = (A1*HO+A1*K1*H1+A2*K*H2+A3*K*H3)/25.4
200 Z = 0.9*HF*(50000/MS)↑(1/3)
210 D = 2*MS*Z*SQR(1+(6.4/Z)↑2)
220 LET W = 9000/D
270 NF = (AP*DS/4)*(AI*T*LI*FI+AF*TF*LF*FF)
280 FOR Y = 1 TO AP  STEP 1
290 N = (NF/AP)*(2*AI*Y/(AI+AF) + Y*Y*(AF-AI)/(AP*(AI+AF)))
295 IF W < 0.025  THEN GOTO 310
300 PS = 1000*N*W*W*W*W*W*W : GOTO 320
310 PS = N/(1000000*(28.672-983.04*W))
320 PT = 2.4455*PS + 8.805*PS*PS*PS
330 PE = (PO - PO/(1+BB*W))*(1 - EXP(-.06*Y))
340 LET P = PO - PT - PE
350 RC(Y) = P*95/PO
360 IF RC(Y) > RL GOTO 420
370 LS = Y - (RL-RC(Y))/(RC(Y-1)-RC(Y))
380 N = INT(N*LS/Y + 0.5)
390 LS = INT(100*LS + 0.5)/100
400 WW = INT(100000*W + 0.5)/100000
410 GOTO 470
420 NEXT Y
430 Y = 0
470 PRINT "T =";HO;"MM";" LS =";LS;"Y.";"  N= ";N
480 PRINT#4,"T =";HO;"MM";"   LS =";LS;"YEARS";"   N =";N;"  W =";WW
490 NEXT HO
500 CLOSE 4
510 DATA 45 : REM  LOWER LIMIT OF PCR
520 REM    MS,  AP,  PO
530 DATA  3500, 25,  7.5
540 REM   AI,    AF,   DS
550 DATA 2000, 10000, 300
560 REM    T ,   LI,   FI
570 DATA 0.050, 0.80, 4.0
580 REM    TF,   LF,   FF
590 DATA 0.060, 0.80, 6.0
600 REM   A1, A2, A3
610 DATA  2,  1,  0.66666667
620 REM    H1,  H2,  H3
630 DATA  100, 150, 200
680 END
```

APPENDIX A, Test Example

T = 50 MM	LS = 3.26 YEARS	N = 344050	W = .03454
T = 82 MM	LS = 5.87 YEARS	N = 695426	W = .03014
T = 114 MM	LS = 9.13 YEARS	N = 1314845	W = .02673

The last line corresponds to the example on Page A-3

Regular Example: W1 = 0.01489"

T = 25 MM	LS = 11.52 YEARS	N = 2389522	W = .02147
T = 50 MM	LS = 13.17 YEARS	N = 2730081	W = .0197
T = 75 MM	LS = 14.56 YEARS	N = 3019934	W = .01819
T = 100 MM	LS = 15.83 YEARS	N = 3282166	W = .0169
T = 125 MM	LS = 17.02 YEARS	N = 3528270	W = .01577
T = 150 MM	LS = 18.16 YEARS	N = 3764855	W = .01479 +
T = 175 MM	LS = 19.27 YEARS	N = 3995293	W = .01392
T = 200 MM	LS = 20.36 YEARS	N = 4221709	W = .01315

Data for regular example: BB = 60

```
510 DATA 55 : REM LOWER LIMIT OF PCR
520 REM    MS,  AP,  PO
530 DATA  5000, 25,  7.5
540 REM   AI,    AF,   DS
550 DATA 6000,  6000, 300
560 REM    T ,   LI,   FI
570 DATA 0.080, 0.80, 3.6
580 REM    TF,   LF,   FF
590 DATA 0.080, 0.80, 3.6
600 REM   A1, A2, A3
610 DATA  2,  1,  0.66666667
620 REM    H1,  H2,  H3
630 DATA  140, 152, 533
680 END
READY.
```

+ T = 150 mm represents the equivalent overlay matching the strength of the original design, because of **W = W1**

Example in Figure 6

The example in Figure 6 of the main body of the paper was calculated with BB = 100 .