

# Ice Road Assessment, Modeling and Management

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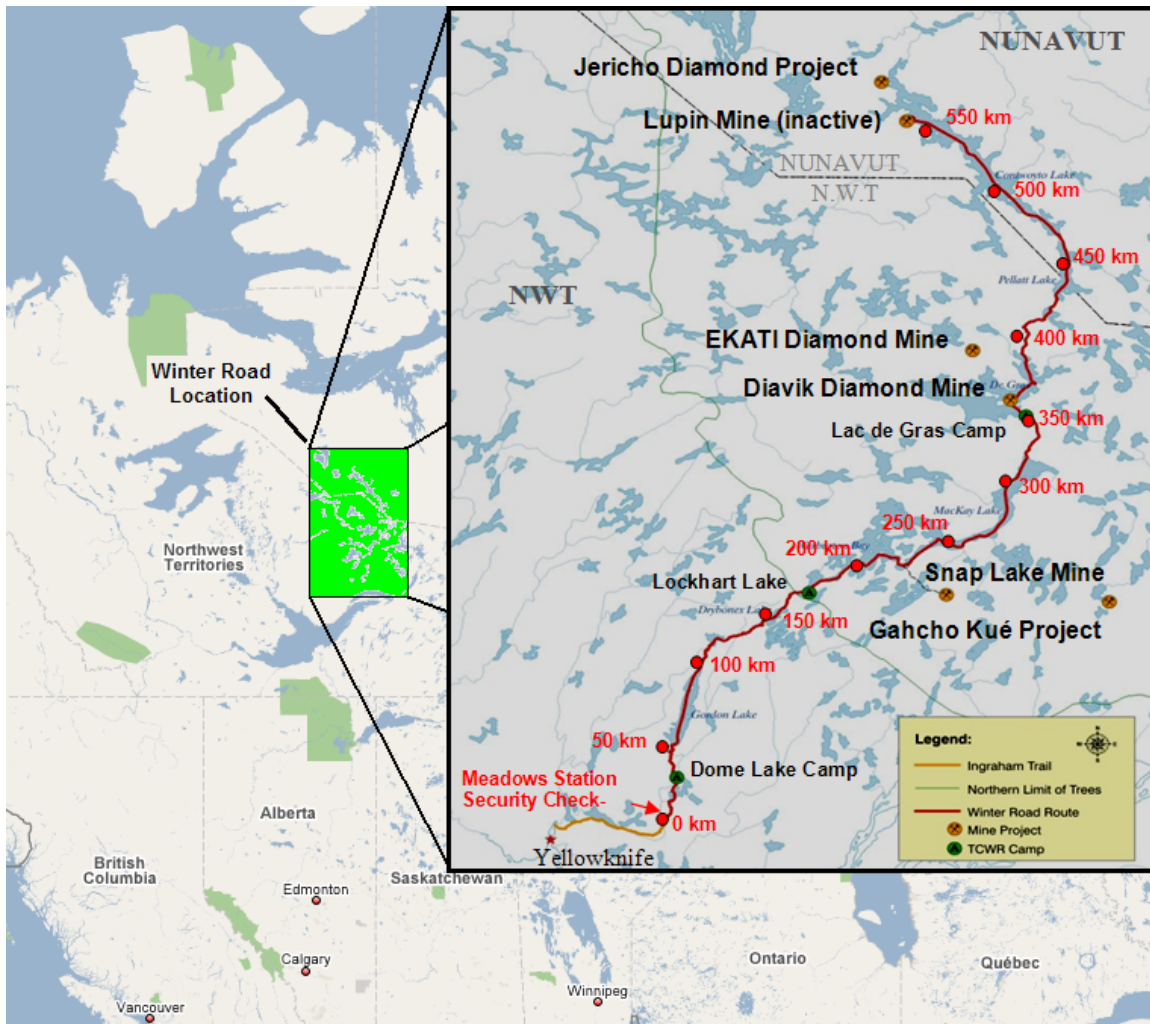
## **ABSTRACT**

The Tibbitt to Contwoyto Ice Road, situated north of Yellowknife in Canada's Northwest Territories is the world's longest heavy haul ice road. This ice road carries more than 8,000 truck loads annually of various axle configurations during a haul season restricted to mid-January through mid-April. Similar to its unfrozen counterpart, the load bearing capacity and ice sheet performance of this road are influenced by the frozen structure and composition and the structure beneath the ice sheet. As global warming trends are expected to continue, the construction and maintenance of these frozen haul routes will become more and more difficult requiring better engineering decisions based on improved information to maintenance personnel and truck haul operator safety. Although the frozen roadway disappears each spring, the effective management of this infrastructure asset is critical to the exploration and mining operations along its route.

This paper presents the recent development of innovative non-destructive ice sheet measurement and assessment technologies allowing comprehensive ice sheet characterization and performance modeling. Research undertaken since 2005 seeks to advance GPR technology, advance the understanding of the relationship between ice dielectric measured at microwave frequencies and its relationship to in-situ ice properties and the identification of internal ice defects. Through these technological advances, it is expected that the availability of this information will allow unprecedented advancement of the science of ice sheet mechanics and allow the modeling of ice growth (quality and thickness) and allow the establishment of enhanced ice sheet loading guidelines to improve safety in variable ice conditions.

## INTRODUCTION

Canada's Arctic Region has long been a source of a variety of base and precious metals. In the 1980s and 90s, exploration was focused on locating mineral deposits that contain diamonds. The discovery of commercially viable diamond deposits in 1991 led to the development of large-scale mining and exploration activities, and unparalleled logistical challenges to support these activities.



**FIGURE 1 The Tibbitt Lake to Contwoyto Lake Winter Ice Road.**

The Tibbitt Lake to Contwoyto Lake Winter Ice Road (Figure 1), situated north of Yellowknife in Canada's Northwest Territories, is the world's longest heavy haul winter ice road comprising overland and over ice crossings. The original winter ice road was established to provide logistical support to the Lupin Gold Mine operated by Echo Bay Mines Ltd. from 1979 to 1998. The current winter road is operated by a joint venture formed between BHP Billiton Diamonds Inc., and Diavik Diamond Mines Inc. Today, this winter ice road typically carries more than 8,000 truckloads annually of various axle configurations during a haul season restricted to

mid-January through mid-April. Similar to its unfrozen counterpart, the load bearing capacity and ice sheet performance of the ice crossing segments of the winter road is influenced by the ice structure and ice composition and the structure beneath the ice sheet.

The winter road provides a route to haul equipment, essential supplies, and fuel to all of the mining and exploration operations located along the route. Since the challenging and environmentally sensitive terrain makes it difficult and prohibitively expensive to construct an all-weather road along the same route, a frozen roadway is created annually to move the resources essential to northern operations. The winter road route is 568 km long, crossing 65 lakes, with 64 overland portages. These land-based portages comprise 73 km (13%) of the total route length. Although the frozen roadway disappears each spring, the effective management of this supply road is critical to the exploration and mining operations along its route. Economic studies for a twenty-year period from 2001 to 2020 estimate that arctic exploration and mining activities will result in C\$12B in direct tax revenues to various levels of government.

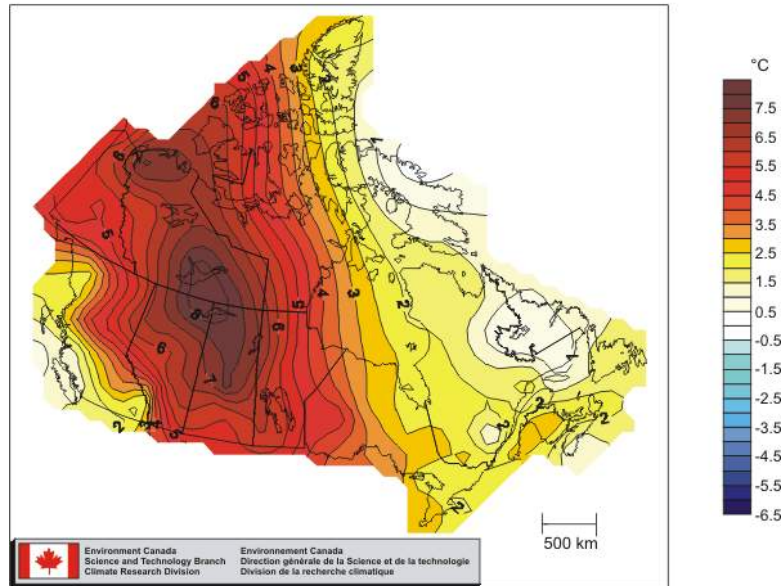
Although the winter road has been operated effectively for more than 25 years, the continuation of status quo operations are no longer acceptable due to changing global warming effects, dramatic increases in haul traffic, and the loss of expertise through retirement of key staff members responsible for maintaining and operating the road. These influences have led to the re-evaluation of the annual construction and maintenance practices of this unique transportation asset. Specifically, the joint venture has funded studies since 2001 to improve the engineering methods for ice sheet quality assurance during construction and operation. These studies have additional significant benefits of improving ice route haul safety through improvements to the science and understanding of ice sheet capacity and vehicle speed recommendations.

## **CHANGING CONDITIONS**

Separate studies on the impacts of global climate change have been completed by Environment Canada and the Arctic Climate Impacts Assessment (1) initiative. Although a detailed analysis of the effects of global warming are beyond the scope of this paper, both studies forecast continuing increases in annual temperatures in Canada's Arctic.

The catastrophic effects of historically unseasonable elevated temperatures were seen during the winter of 2005/2006. Environment Canada reported winter temperature increases above normal in western/northern Canada of more than +7°C resulting in an annual temperature increase of +4°C centered geographically over the winter road location as shown in Figure 2.

**TEMPERATURE DEPARTURES FROM NORMAL**  
**Winter (Dec, Jan, Feb) 2005/2006**  
**ANOMALIES DE LA TEMPERATURE PAR RAPPORT A LA NORMALE**  
**Hiver (dec, jan, fev) 2005/2006**



**FIGURE 2 Temperature Departures from Normal Winter (Dec., Jan., Feb.) 2005/2006.**

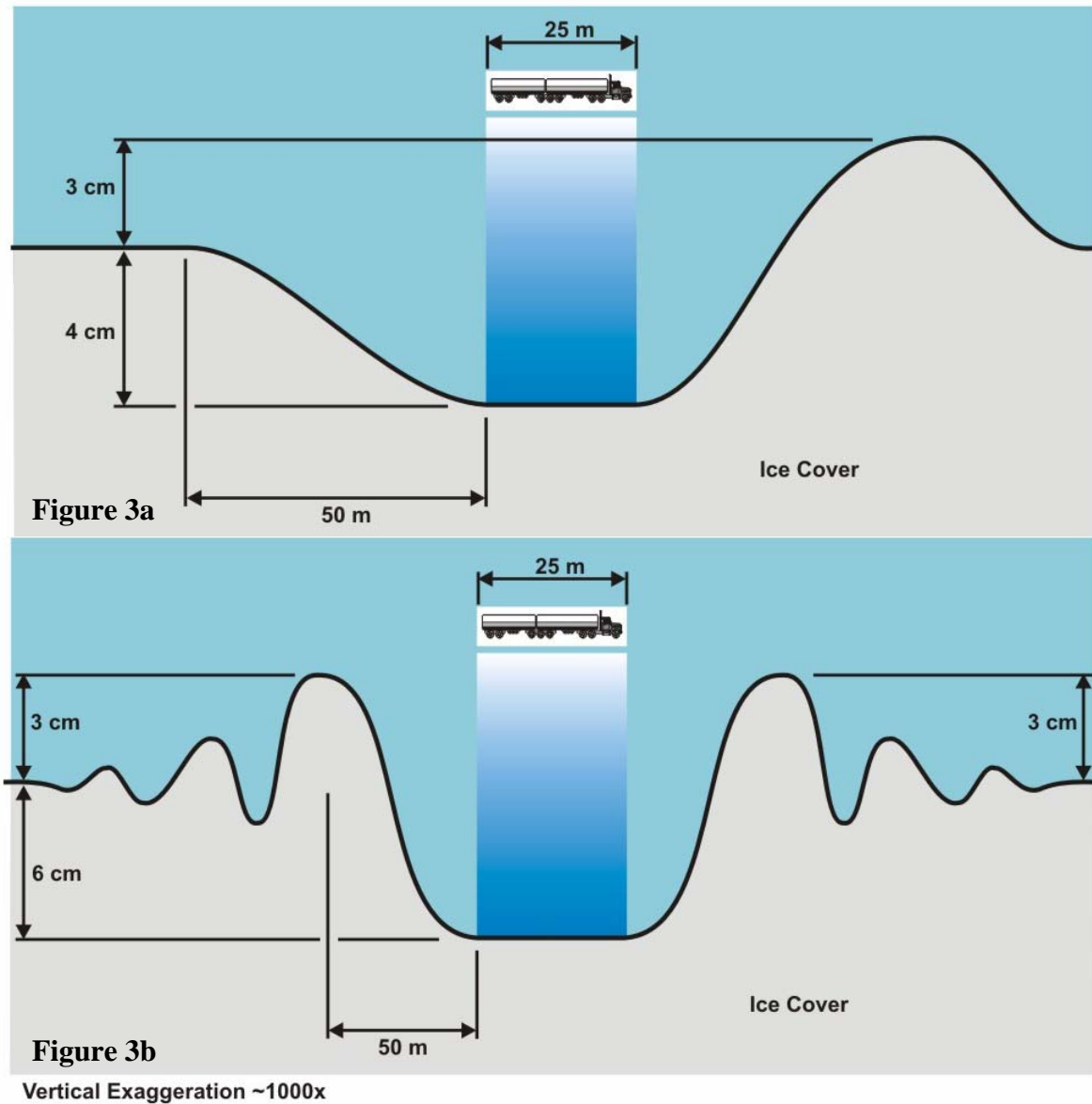
Due to this extraordinarily warm winter season, the winter road was only open for 50 days, never attaining full load capabilities, and closed early due to deterioration of the ice road and open water conditions. Compounding the effects of the shortened haul season is the dramatic increase in haul traffic for the winter road. The haul traffic statistics for the past decade are summarized in the table below. The increased traffic and the climate induced early closure of the winter road operations reconfirmed the necessity for enhanced management of the roadway asset.

**TABLE 1 Winter Road Traffic Statistics**

Year	Truck Loads (north bound only)	Tonnes of Freight	Date Road at B-Train Capacity	Date Road at Super B Capacity	Date Opened	Date Closed	Days Open
1996/1997	3,500	100,000					
1997/1998	2,543	82,000					
1998/1999	1,844	57,000					
1999/2000	3,703	125,380			January 29	April 3	65
2000/2001	7,981	222,712	February 25	March 3	February 1	April 13	72
2001/2002	7,735	245,586	February 14	February 22	January 26	April 16	81
2002/2003	5,243	198,818	February 16	February 21	February 1	April 2	61
2003/2004	5,091	179,144	February 17	February 23	January 28	March 31	64
2004/2005	7,607	252,533	February 13	February 20	January 26	April 5	70
2005/2006	6,841	177,674	March 6	Never	February 5	March 26	50
2006/2007	10,922	330,000	February 19	February 26	January 28	April 9	72

## ICE DYNAMICS AND BEARING CAPACITY OF ICE COVERS

Ice floats because the density of ice is less than the density of liquid water. Hydrostatic pressure on the bottom of the ice, or its buoyancy, is the primary source of bearing capacity for a floating ice sheet. This buoyancy force supports the ice sheet which in turn deforms to distribute the load evenly over a large area. In the case of vehicle traffic, a vehicle moving across an ice cover creates a dynamic load-deflection bowl that moves with it (as shown in Figure 3a), continually displacing the water in a manner similar to a shallow draft boat. Like a boat, the moving deflection bowl generates waves in the ice sheet in front of and behind the load that travel at speeds that are dependent on the localized ice structure and water depth.



**FIGURE 3 Ice Sheet Deflection Below and Above Critical Speed.**

The ice sheet stresses are amplified as the vehicle approaches the ice sheet wave speed, referred to as the critical speed (Figure 3b above). The critical speed for a particular ice sheet depends on the water depth and structural parameters of the ice. These parameters include ice thickness, ice stiffness (the effective elastic modulus), Poisson's ratio of the ice, type of ice (black ice or bubble rich white ice), structural homogeneity, and structural defects. Increases in ice thickness or water depth result in increases in the critical vehicle speed, allowing faster haul speeds and correspondingly shorter haul times - important to ice road performance, efficiency and safety. Conversely, variations in ice type (and the corresponding changes in material properties such as elastic modulus, Poisson's ratio or strength) or the presence of structural defects can reduce the critical speed. The effects of these variations are discussed in detail in the U.S. Army Corps of Engineer's Cold Regions Research and Engineering Laboratory Ice Engineering guidelines (2).

Traditionally, floating ice covers have been modeled as uniformly loaded elastic, homogeneous, and isotropic plates resting on an elastic foundation. In such models (3), failure is defined as the point at which flexural stresses in the plate (ice cover) exceed an allowable stress and cracking occurs. The main difficulty in using these models is the selection of an appropriate allowable ice stress and the conditions under which the ice cover meets the theoretical assumptions. Extensive research (4, 5, and 6) has developed ice sheet bearing capacity equations for short-term loads based on empirical results for a wide variety of ice strengths. Under the assumption that an ice sheet is homogenous, isotropic and free of structural defects, the simplified equation is given in Equ. 1.

$$P = Ah^2$$

Where:  $P$  = allowable load

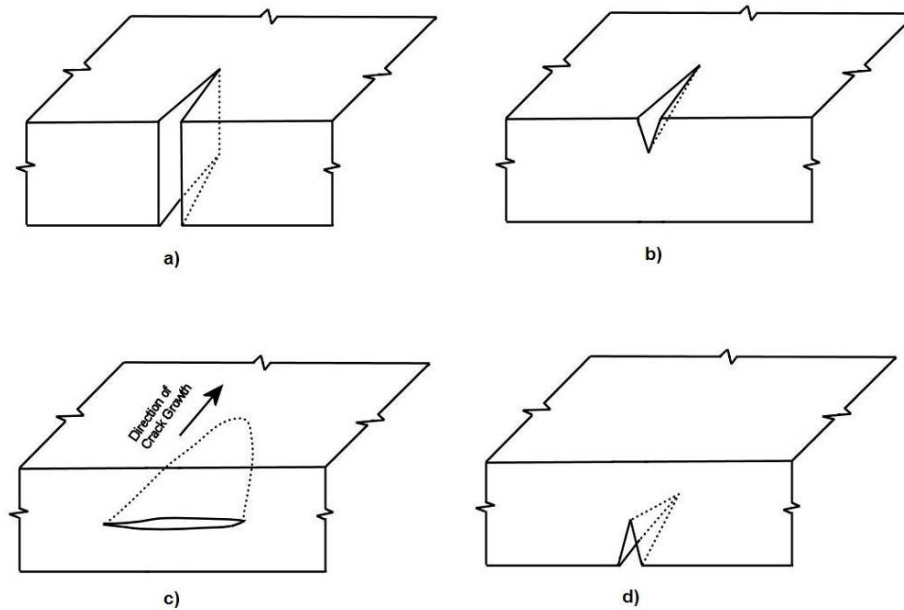
$h$  = effective ice cover thickness

$A$  = coefficient dependant on ice quality (strength), ice temperature, load geometry and desired risk level.

### EQUATION 1

For practical and imperfect ice sheets, structural defects of interest for ice sheet quality assessment impacting bearing capacity calculations are depicted in Figure 4. These defects include a) vertical full depth wet cracks that propagate horizontally, b) and d) vertical cracks that propagate vertically and may develop into full depth cracks, and c) horizontal cracks that propagate within the ice sheet and can result in structural delaminations or spalling of the ice sheet.





**FIGURE 4 Ice Sheet Defects.**

Currently for the Tibbitt to Contwoyto Winter Ice Road, a Wyman's stress analysis approach (3) is used to estimate the required ice thickness for short-term moving loads. During this analysis, ice cover stresses for a given vehicle footprint and axle load distribution are compared with allowable ice stress limits defined by the in-situ ice structure. Either back calculated or measured in-situ ice strengths can be used to evaluate the allowable vehicle loads. Wyman's approach also allows the analysis of dynamic wave effects and their impact on ice stresses and deflections.

In order to maximize operator safety and ice haul road asset utilization (maximum safe load capacity and maximum safe haul speeds), the continuous assessment of route bathymetry (water depth), effective ice thickness, and the presence of ice structure defects is required. The challenge has been to accurately determine the ice thickness and estimate the ice quality in a cost effective and timely manner. U.S. Army Corps of Engineer's Ice Engineering (2) guidelines for the assessment of both parameters suggest that the ice sheet characteristics should be determined manually every 50 metres or more frequently in variable ice conditions. For ice roads of even moderate lengths, this testing frequency is not practical with current destructive manual auger based measurement methodologies.

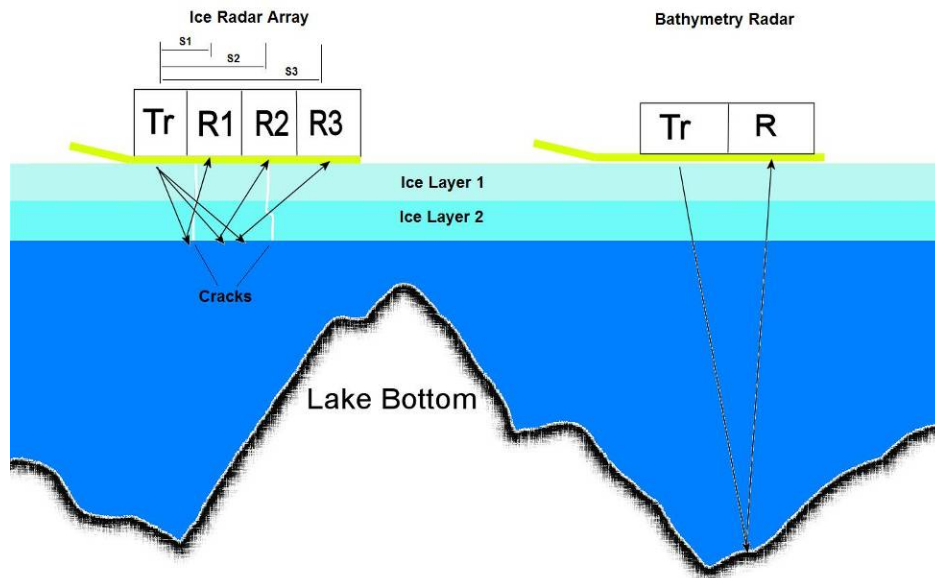
## **ICE AND BATHYMETRY MEASUREMENTS**

As discussed, the load carrying capacity and critical vehicle speed of an ice haul road is a function of ice thickness, ice material properties, and localized bathymetry (water depth). In order to provide this crucial information to ice sheet engineers, EBA adapted its **ROAD RADAR™**

ground penetrating radar (GPR) technology for ice and bathymetry measurements. The **ICE ROAD RADAR™** system has two separate and synchronized radar systems integrated with linear and DGPS based spatial referencing subsystems.

The ice sheet measurement radar determines both the signal travel time and ice dielectric values (calculated from pulse propagation velocity) at every measurement point for each detected ice sheet layer. The ability to measure variations in ice properties makes the system self-calibrating for ice thickness measurements and eliminates the requirement for augured hole-based calibrations. These capabilities are achieved through the use of a high resolution multi-channel surface coupled antenna array based radar system. The system operates at a frequency of 1.1 GHz and uses a single transmit antenna (Tr) and three receiver antennas (Rn) positioned at precise separations (sn). The radar system can acquire samples at any programmed distance (typically every 100 mm at 80 km/h) and can resolve ice layers as thin as 100 mm to a nominal maximum depth of 2 m dependent upon ice sheet properties.

The integrated bathymetry radar system uses a conventional bi-static radar configuration that is also surface coupled to minimize surface reflection losses. The bathymetry system operates at 200 MHz and uses a single transmit antenna (Tr) and receiver antenna (Rn) pair. This radar system acquires bathymetry measurements typically every metre and has been designed to measure to a maximum water depth of 10 m. The schematic for the radar systems are shown in Figure 5.



**FIGURE 5 ICE ROAD RADAR™ System Schematic.**

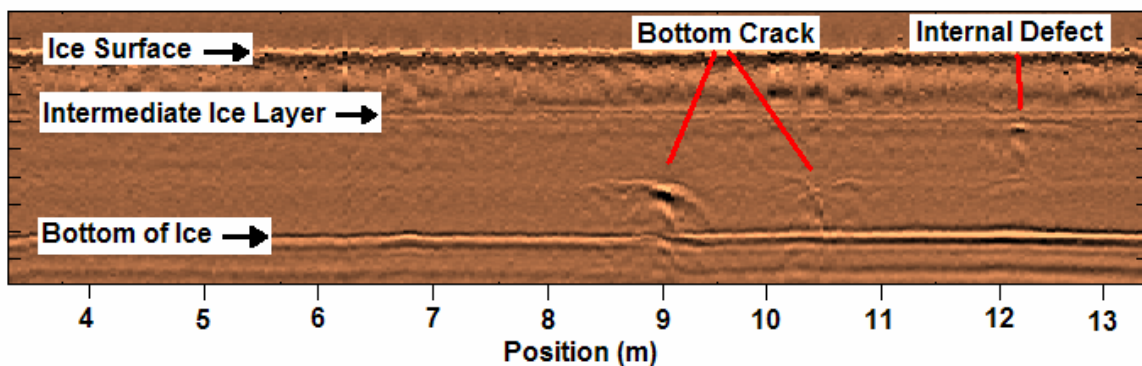
The **ICE ROAD RADAR™** system vehicle, including ice radar system, bathymetry radar system, DGPS and digital video system is shown in Figure 6. The surface coupled ice radar array and

bathymetry radar can be seen on the high molecular weight plastic towing surfaces behind the vehicle. This vehicle can travel at speeds up to 80 km/h while conducting ice sheet surveys.



**FIGURE 6 ICE ROAD RADAR™ System Survey Vehicle.**

Due to the unique surface coupled ice antenna array and the measurement recording frequency used for continuous sampling, the system has the ability to measure ice material properties variations, ice thickness variations and map internal ice structure defects (such as cracks and voids) previously not possible. EBA ice sheet engineers speculate that the radar dielectric, as measured, will provide the ability to identify homogeneous ice sheet sections and perhaps, more importantly, quantify variations in material properties affecting ice sheet strength due to entrained (trapped) air content, suspended impurities, or intercrystalline unfrozen water.



**FIGURE 7 Sample Raw Ice Array Data.**

During limited field trials on the winter road of the **ICE ROAD RADAR™** system in March 2007, ice dielectric measurements were found to vary by more than 15% between black (low air) and

white (high air) ice. Sample data from a single radar channel collected at a 20 mm sampling resolution during those trials is shown in Figure 7. This data shows intermediate structural layers within the ice sheet of differing ice types, and a variety of structural defects (bottom cracks and an internal void) previously undetectable.

Although not previously detected in ice sheet radar data, these intermediate structural layers are consistent with ice road construction practices. While snow clearing is used to increase natural ice growth rates, flooding is used to quickly increase localized ice thickness (analogous to asphalt overlays and patches on paved roadways). Figure 8 shows a typical flood crew at work. These flooded areas then have layered ice structures, with each layer having potentially different ice properties.



**FIGURE 8 Ice Flooding Activities to Increase Ice Sheet Thickness.**

## **ASSET MANAGEMENT**

The Tibbitt Lake to Contwoyto Lake Winter Road was first created in 1979 and has been constructed and maintained by Nuna Logistics Limited annually since 1998. Unlike conventional roadways, the seasonal loss and subsequent reconstruction of an ice road allows the refinement of the alignment based on the performance of the road in previous seasons and any significant visual defects such as cracks or ridges. Historically, the annual road alignment was established by senior operations/maintenance personnel based on personal experience. Once the alignment was defined, the route was flown and marked by helicopter following freeze-up (typically mid-December) and the clearing of surface snow and flooding of the ice sheet was started to accelerate ice growth. A consistent nominal ice sheet thickness of 0.7 metres is targeted to allow the opening of the road.

Historically, a core group of maintenance contractor professionals developed and maintained road-specific expertise to effectively manage this unique roadway asset every season. This expertise included improving methodologies for locating the roadway alignment, the ongoing monitoring of ice thickness, and the analysis of repeatedly problematic areas. Operational information such as ice thickness data and problem ice areas was collected frequently and acted upon immediately during the active construction and maintenance season adding to the expertise associated with the operation of the ice road. This growing expertise was enhanced and propagated from one year to the next through the involvement of the core Nuna Logistics staff members. In 2001, a high precision GPS based centre-line survey was conducted to accurately document the road location and provide a foundation for future asset management. This survey was repeated in 2004 to capture any changes or improvements to the alignment since 2001.

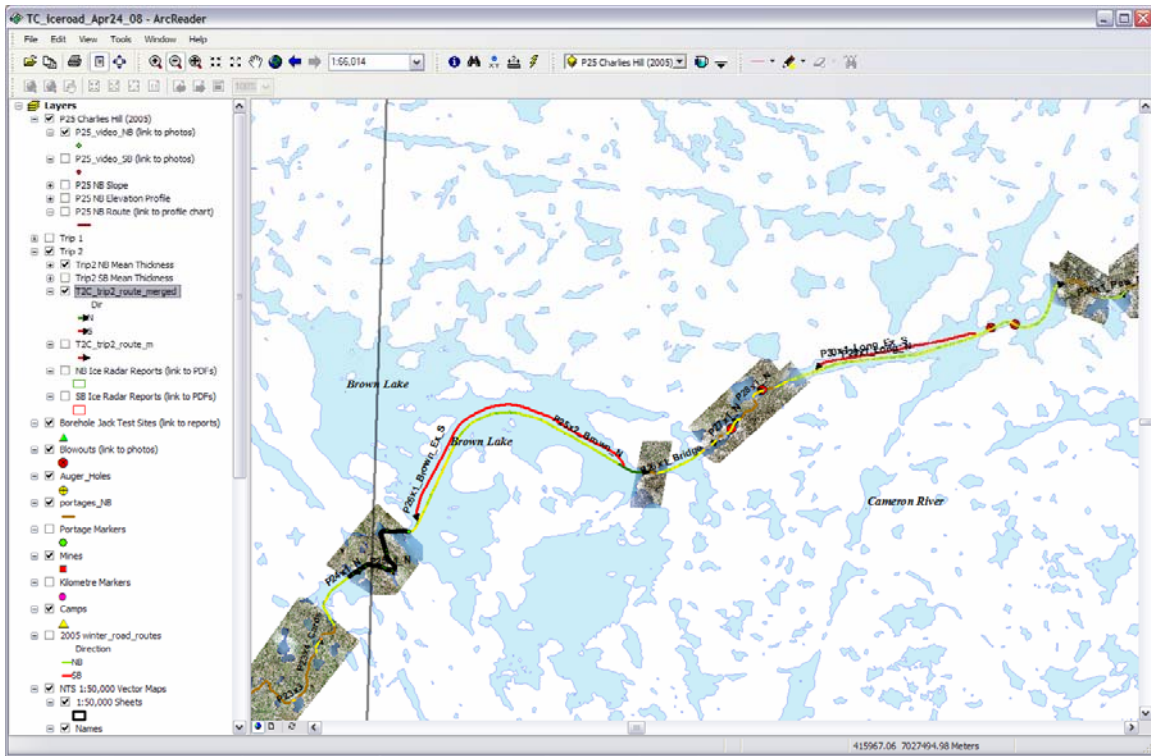
During the 2005/2006 season, the extraordinary warm winter experienced in Canada's Arctic introduced unprecedented conditions for constructing and operating the winter road. These conditions, which will be further exacerbated by global warming, and the loss of onsite expertise through retirement, present new challenges that force the consideration of new approaches to assist in the management of this unique roadway asset. It was recognized that a total asset management approach to the Tibbitt to Contwoyto Winter Road could help maximize the serviceability of the road, while minimizing the costs associated with its annual maintenance. The framework proposed for the 2008/2009 season includes:

- spatial winter road route information (lake boundaries, portage information, route elevation data);
- detailed spatial roadway alignment(s) (including all available previous years) maintained within a GIS environment;
- detailed spatially referenced radar based bathymetry information (including all previous historical surveys);
- a repository for significant roadway maintenance activities;
- repository for all road monitoring data (radar measured ice thickness, augered hole ice thickness, in-situ ice strength measurements, documented or known ice failure locations);
- radar based ice parameter measurements (ice layers, ice material properties (dielectric), structural defects) temporal and location referenced; and
- climatic conditions (ambient air temperature).

The development of this GIS based asset management framework (Figure 9) will allow the accurate monitoring of ice growth rates based on lake characteristics (location, turbidity, depth, currents, etc.), in-situ strength measurements, maintenance activities (flooding), and climatic conditions. Of perhaps greater interest is the analysis of the ice structure from a load bearing capacity given the number, dielectric property, type and thickness of detected ice layers. The accurate referencing of the radar measured thickness and dielectric parameters will allow the confident monitoring of the same precise locations throughout the season. It is expected that this capability will allow the development of local ice growth models for future years based on



climatic and local lake conditions. More importantly, it is expected that this asset management approach will help engineers optimize load capacity and haul vehicle speeds based on ice properties and water depth information not previously available or considered.

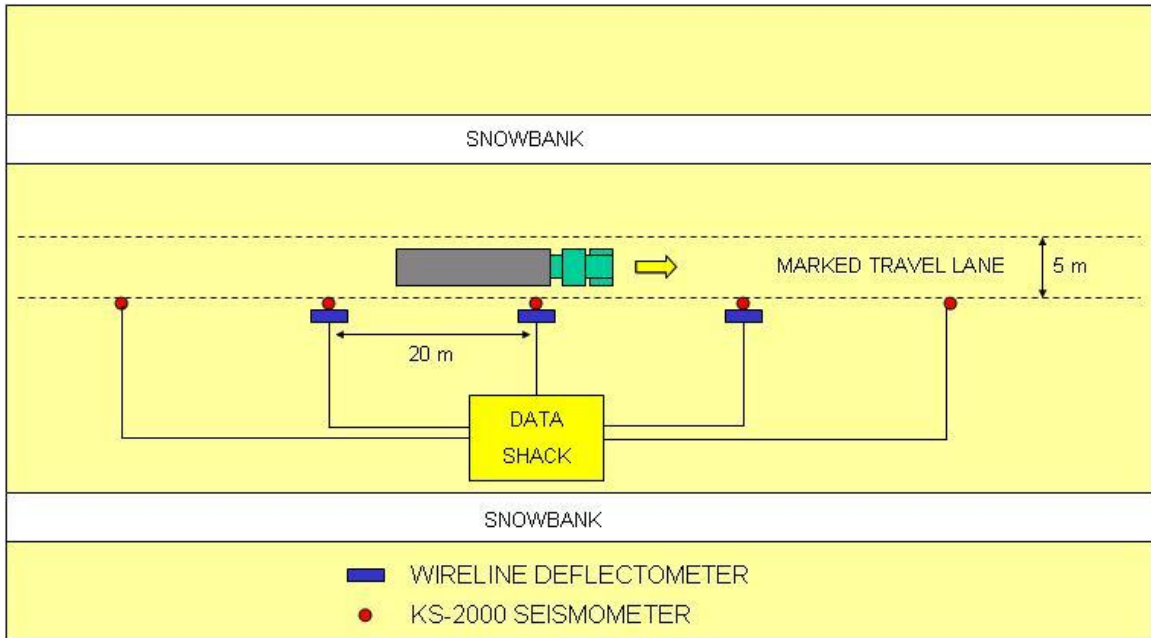


**FIGURE 9 GIS Based Asset Management Framework for the NWT Ice Road.**

## ONGOING ICE SHEET PARAMETER VALIDATION AND VERIFICATION

Each ice road maintenance season allows the continuing verification of in-situ ice sheet structural parameters through field trials combined with the collection of extensive field ground truth data. To date, these structural parameters have included ice sheet deflection, in-situ borehole jack based compressive strength, and radar based structural and ice property measurements.

In 2005, the analysis of modeled ice deflection parameters was undertaken using a combination of wireline displacement and vertical acceleration seismometer transducers. These transducers were situated in a linear array on a lake along the Tibbitt to Contwoyto Ice road route. The ice thickness and bathymetry information was collected at the time of the trials allowing dynamic ice sheet deflection as a function of vehicle speed to be modeled. Field trials were then undertaken using a loaded truck at various speeds to cause the ice sheet surface to deflect. The field trial test configuration is shown in Figure 10, and the truck used during the trials is shown in Figure 11.

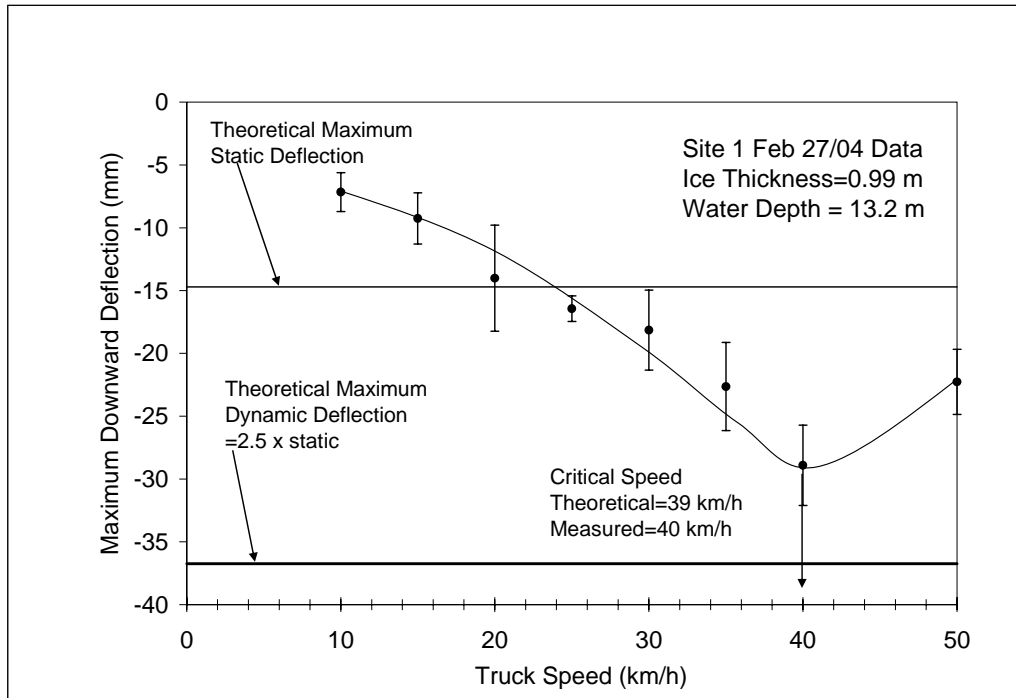


**FIGURE 10 Ice Deflection Field Measurement Configuration.**



**FIGURE 11 Ice Deflection Field Trials.**

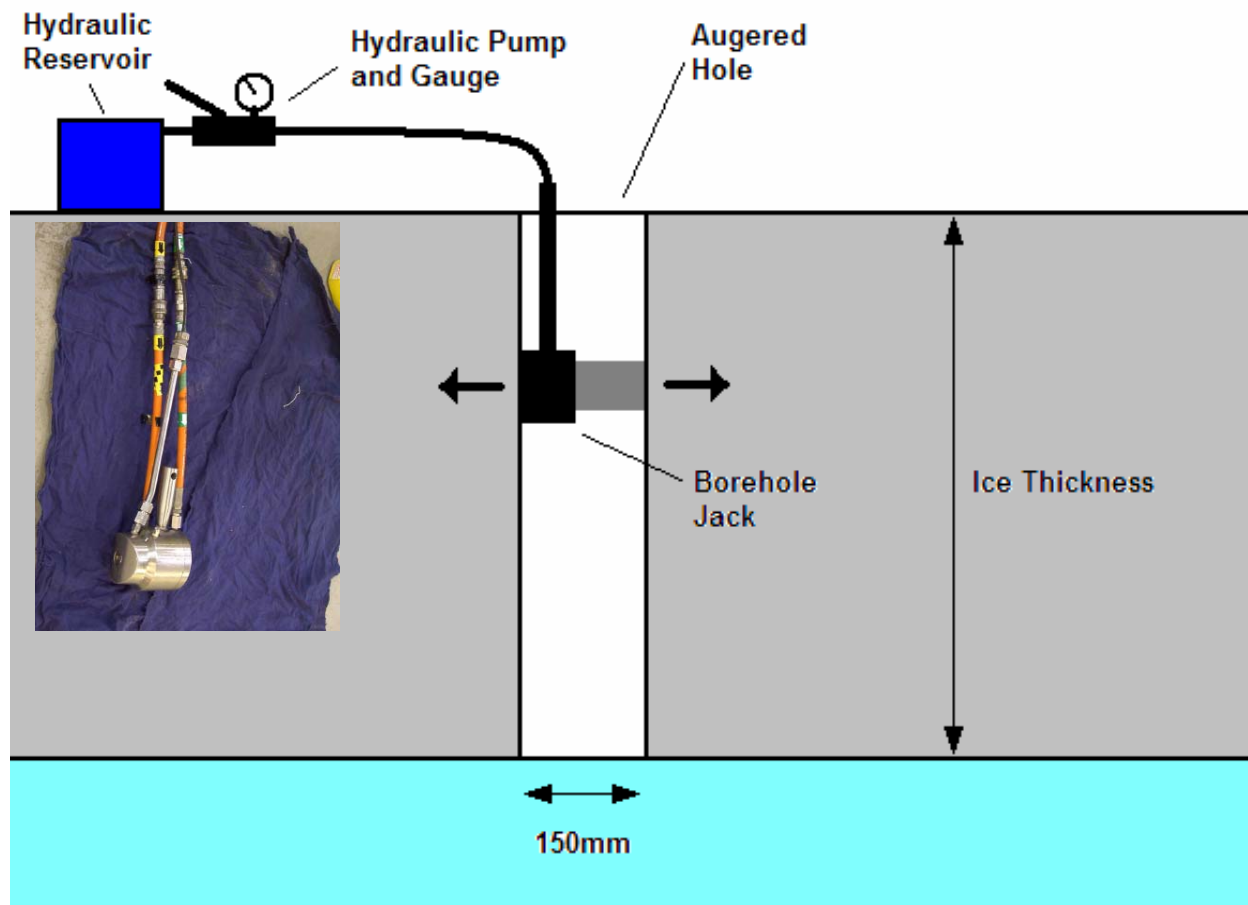
Results from the ice deflection study are presented in Figure 12, showing that maximum ice deflection increases with vehicle speed, reaching a maximum at 40 km/h. These results confirm the theoretical relationship between in-situ ice structure, water depth, the resulting ice deflections and vehicle speed. Speed limits must be imposed on heavy haul ice roads to prevent overstressing the ice at speeds approaching critical speed. Appropriate speed limits will depend on ice properties, ice thickness, water depth and vehicle loads.



**FIGURE 12 Ice Deflection Results.**

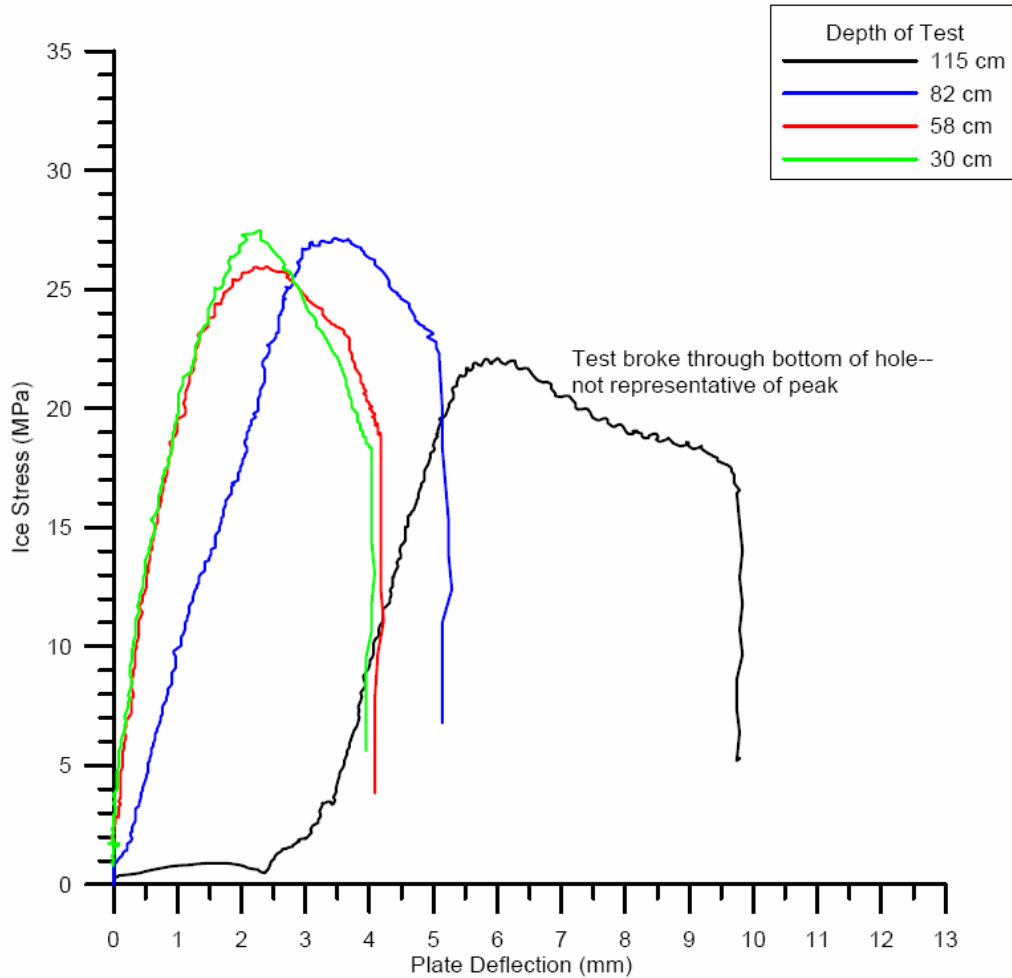
The 2007/2008 winter season included field data collection using destructive augered holes, as well as non-destructive **ICE ROAD RADAR™** data. The auger holes were used to confirm water depth, ice thickness, and in some cases were used to measure the in-situ confined compressive strength of the ice at various depths within the ice sheet using a hydraulic borehole jack. For these strength tests, a borehole jack is placed in the augered hole and is expanded horizontally against the sides of the hole until failure is reached. During each test sequence, displacement and pressure parameters were recorded. The schematic for the borehole jack compressive test is shown in Figure 13.





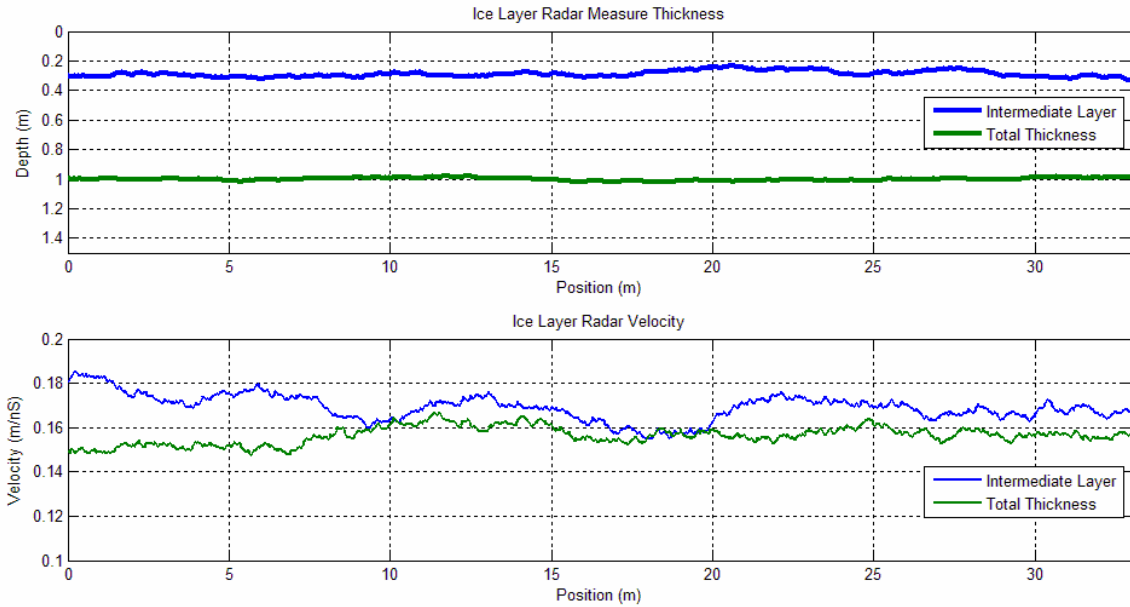
**FIGURE 13 Borehole Jack Test Methodology.**

Figure 14 shows an example of in-situ compressive strength test results at depths ranging from 30 to 115 cm within the ice sheet. This figure depicts the relationship between horizontal deflection (jack expansion) and borehole wall stress (force per unit area) at each depth. It is believed that compressive strength tests from borehole jack measurements will prove to be an important quantitative ice property measurement methodology capable of determining localized in-situ ice strengths at specific depths within the ice sheet. These strength measurements will be compared to the radar measured ice property variations and ice types encountered along the road.



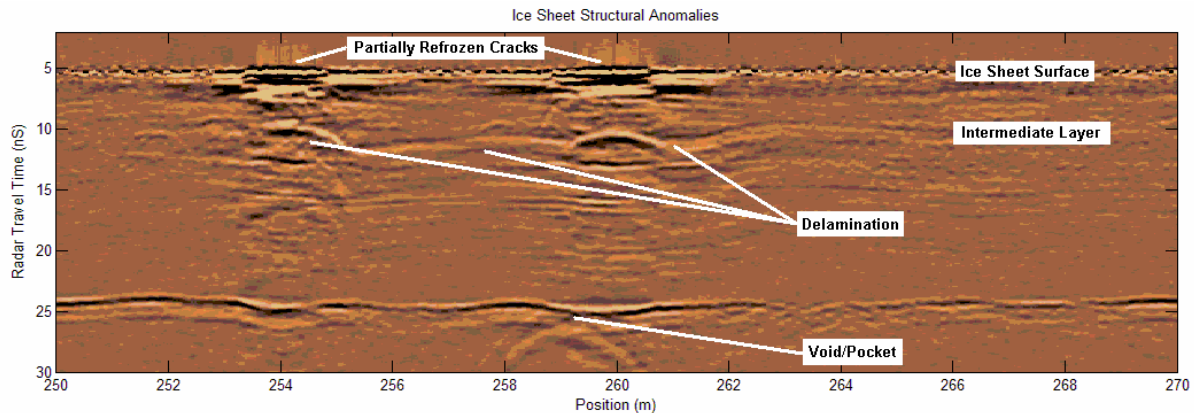
**FIGURE 14 Borehole Jack Test Results.**

Also during the 2007/2008 winter season, the **ICE ROAD RADAR™** system was used to collect multi-channel radar data which allows the calculation of the ice layer thickness and radar based electrical properties of the ice sheet (Figure 15) and the analysis of localized ice structure defects and anomalies (Figure 16).



**FIGURE 15 ICE ROAD RADAR™ Test Results.**

These data will allow the investigation of possible relationships between radar measured ice sheet dielectric parameters and the in-situ ice strength and will include radar measured ice structure (individual layer thickness, layer type, and any internal defects) and radar measured dielectric properties for each layer.



**FIGURE 16 ICE ROAD RADAR™ Detected Structural Anomalies.**

## CONCLUSIONS

The load bearing capacity models currently used to predict the ice sheet performance for the Tibbitt to Contwoyto Ice Road are based on simplifying assumptions for the ice structure and the region beneath the ice sheet. Historically, the minimum ice thickness required for the targeted bearing capacities could be routinely achieved based on local experience and careful ice sheet maintenance and construction practices. During the winter of 2005/2006, climatic anomalies in

Canada's north made the ability to attain the minimum ice thickness impossible. This climatic event, combined with the potential loss of significant operational expertise through retirement, demonstrated the necessity for a comprehensive asset management approach.

This paper presents the recent development of innovative non-destructive measurement and assessment technologies and a GIS based framework for a comprehensive asset management system. This asset management system will provide a repository for the considerable historical operational and performance information and enhanced ice structural parameter and bathymetry data presented in this paper. It is believed that this in-situ data will allow more accurate ice characterization leading to refinements in the bearing capacity models and improved maintenance personnel and truck haul operator safety.

Detailed analysis of data collected during the winter 2007/2008 survey season is currently underway. This analysis is focused on understanding the relationship between ice dielectric measured at microwave frequencies and its relationship to in-situ ice properties and in-situ strength. Radar analysis techniques are also being developed to identify internal ice defects, lake bottom features, and quantify water depth. The analysis will continue through an expanded 2008/2009 field program.

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