A STUDY OF THE PATTERN OF HEADWAYS ON AN URBAN FREEWAY

la Voirie
Québec
da

Jean-Luc Simard, B.A, Ing.P, B.Sc.A, M.Sc.A.
Ingénieur en circulation

FORM TO BE BOUND WITH TWO COPIES OF DOCTORAL THESES AND [redacted] OF MASTERS' THESES FACING FRONT COVER.

THE UNIVERSITY OF TORONTO LIBRARY

MANUSCRIPT THESIS

AUTHORITY TO DISTRIBUTE

AUTHOR: This thesis may be lent or individual microfilm copies made available:

(signature of author in *one* of the three spaces below)

(a) without restriction ....Jean Luc Simard.......

OR (b) with the restriction that, for a period of five years (until ....) the written approval of the head of the graduate department is required. ....

OR (c) with the restriction that, for a period of five years (until ....) the written approval of the author is required. ....

BORROWERS: The borrower undertakes to give proper credit for any use made of the thesis, and to obtain the consent of the author if it is proposed to make extensive quotations, or to reproduce the thesis in whole or in part.

SIGNATURE OF BORROWER	ADDRESS	DATE

A STUDY OF THE PATTERN OF HEADWAYS ON AN URBAN FREEWAY

by

J. L. SIMARD

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Applied Science in Civil Engineering
in the Faculty of Civil Engineering,
at the University of Toronto, Ontario

August, 1962

ACKNOWLEDGMENTS

The author of this thesis is very much indebted to the Canadian Good Roads Association whose sponsorship enabled him to carry out this study and to the Quebec Department of Roads, especially their Chief Engineer, Mr. Arthur Branchand, and their Chief Traffic Engineer, Mr. Henri Perron, for granting him a full year leave of absence in order that the work might be completed.

Special acknowledgments are due to: Professor M. M. Davis who as director of this project provided much valuable information and assistance and also reviewed the thesis; Mr. W. Q. Macnee, the chief Traffic Engineer of the Department of Highways of Ontario, for supplying the photographic equipment and the films from which the data involved in this thesis have been extracted; Mr. J. L. Vardon, Planning Engineer in the same Department, for his comments and assistance; Mr. R. Wormleighton, professor in the Faculty of Mathematics at the University of Toronto, for his advice in statistical matters; the author's wife, Suzanne, for her invaluable assistance in extracting the data from the films.

TABLE OF CONTENTS

Table of Contents
continued

LIST OF TABLES

LIST OF FIGURES

Figure No. | Page

List of Figures
continued

I. INTRODUCTION

The purpose of this thesis was to study the longitudinal distribution of vehicles in the traffic stream. In respect of roads with small volumes of traffic, we knew that a good approach to the problem may be obtained by assuming that the traffic is fortuitously distributed and follows closely enough a quite simple probability function named Poisson distribution (3).

However, because of the nature of the Poisson formula and the way it has been derived, we had serious doubts that it would not work as well for average or relatively high traffic volumes. Since all the traffic theory regarding the longitudinal pattern of the traffic flow is based on this function, we thought that it was very worthwhile to check its accuracy at different volumes and possibly determine the conditions for which it applies.

At high volumes, the traffic is not as fortuitously distributed as in the case of low volumes since there are so many factors influencing the driver operation that we no longer have this complete randomness upon which the Poisson function is based. In these conditions, it was felt that another mathematical tool was needed which was not based on this "complete randomness". This tool should possibly work for either the opposite extreme from randomness, which is regularity and can be nearly approached during congestion, or for the whole range of possibilities between randomness and regularity.

A. K. Erlang (5) has developed such a distribution and used it to describe telephone traffic. The Erlang distribution is "less random" than the Poisson distribution in the sense that it predicts a more regular and determined flow than the latter. Since we expected to observe a relative regularity of the traffic flow at high volumes, the Erlang function might possibly work. Furthermore, there might be such an analogy between road traffic and telephone traffic that we could possibly apply to road traffic the theory as developed in telephone by A. K. Erlang.

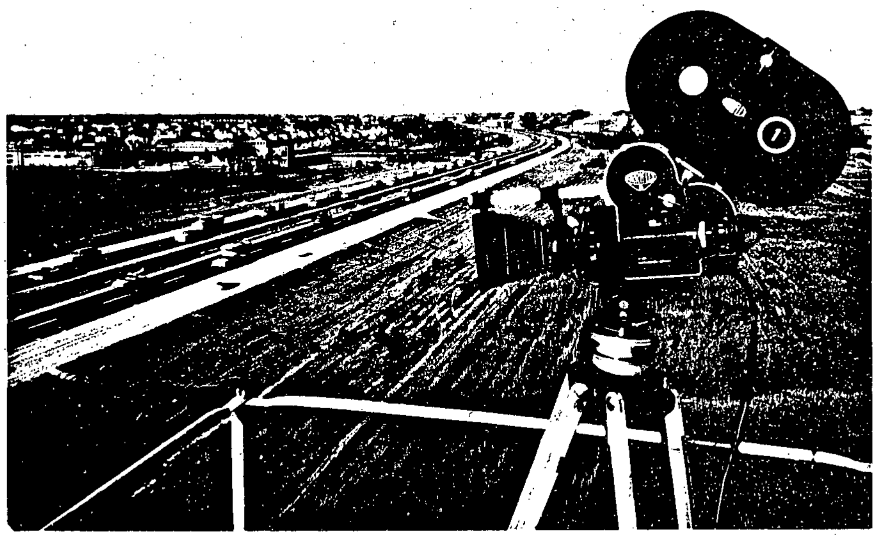
II. EXPERIMENTAL PROCEDURE

THE FIELD EQUIPMENT

The data upon which we based our calculations have been extracted from some films taken by the Dept. of Highways of Ontario in previous studies.

The camera used to take the films was an "Arriflex" 16 m.m. professional cine camera, see Fig. No. 1. Although it was equipped with wide-angle, telescopic and normal lenses, only the normal lens was used. It was also powered by a storage battery and the frame speed could be set at any given speed by a quick adjustment. A small amount of experimenting showed that a speed of eight frames per second would give the best results as far as the number of vehicles observed per length of film and the movement of a particular vehicle in any one frame are concerned.

The film spools contained approximately 400 feet of film which produced approximately thirty minutes of film without changing spools. Although it would have been desirable to have no interruptions in filming during this period, it was occasionally necessary to stop the camera to make speed adjustments. Filming was also stopped if traffic came to a stop either on the through lanes or on the deceleration lane. This stop condition contributed



Arriflex 16 mm. camera used
to obtain the data

FIGURE 1

nothing to the study and in all cases the cause was beyond the camera range and therefore of no interest in this study.

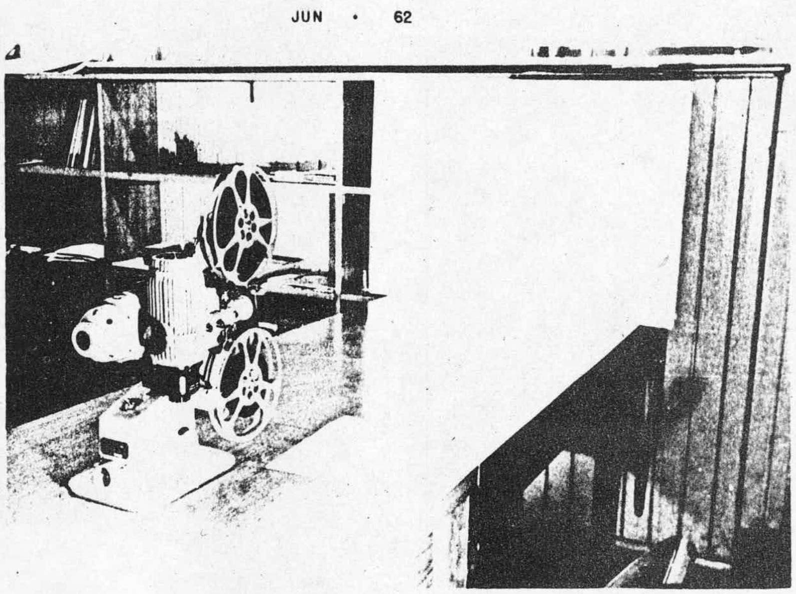
To obtain the required field of view, the camera was located on a tower constructed of portable tubular steel scaffolding (Fig. No. 2). The tower was approximately fifty feet high and guys were used to minimize swaying. The distance between the tower and the nearest lane of the highway was in the vicinity of 100 feet. The tower was plainly visible to the motorists, particularly when it was manned. In order to minimize any effect on the habits of the drivers, it was in place approximately one week before any camera work was carried out. Personnel also moved around at the top of the tower during this familiarization period. As far as could be determined, the tower had no effect on the operating characteristics of the vehicles on the through lanes or on the ramp.

THE PROJECTION EQUIPMENT

The projector used for this phase of the study was a Bell and Howell 16 m.m. silent time and motion study type (see Fig. No. 3). The film could be run forward, reversed or stopped at any time. The projector was specially fitted for manual frame advancement for detailed examination of each individual frame of the film. A counter

Tower from which the films were taken

FIGURE 2



Projector used to extract data
from the films

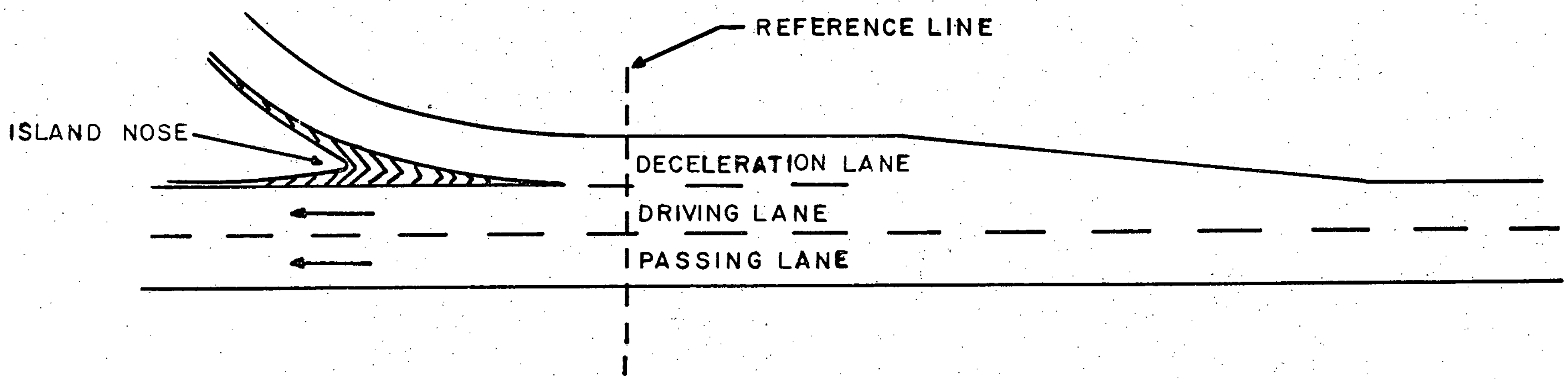
FIGURE 3

was attached to the projector for recording the number of frames which had been viewed between the passage of successive vehicles. The projector was equipped with a normal projection lens with a focal length of 2 inches and required a projector distance of about 20 feet to obtain a picture size of 4 x 3 ft.

THE STUDY LOCATIONS

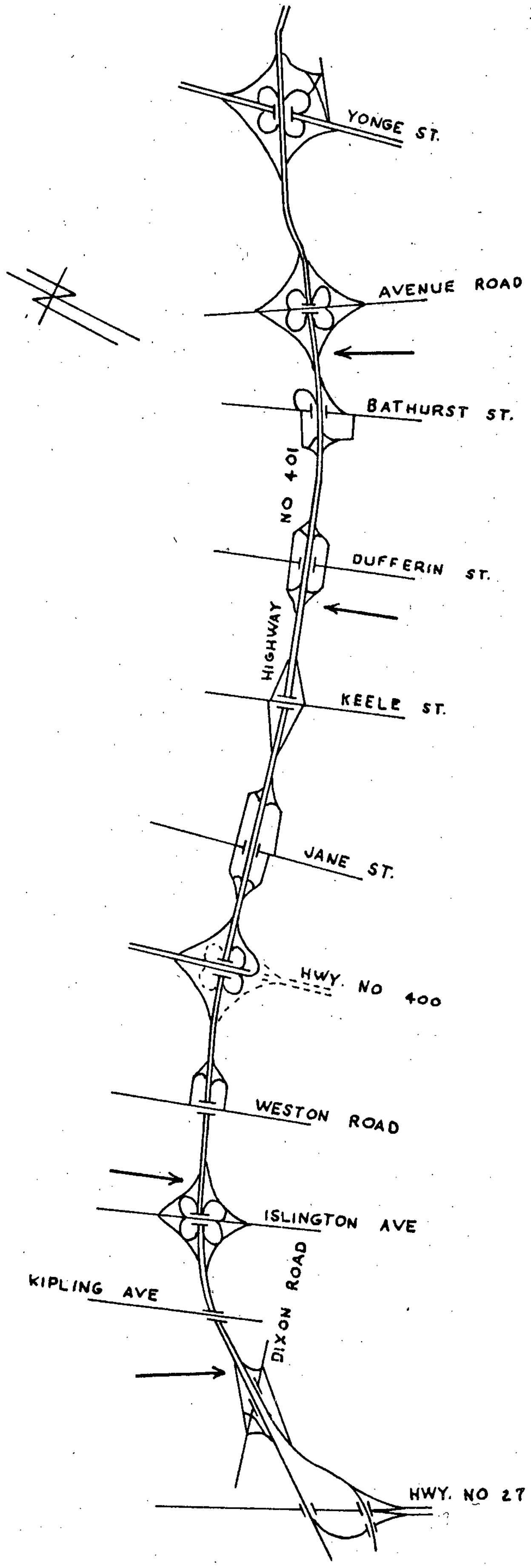
Although the Dept. of Highways of Ontario had a great number of films available which had been taken in previous studies, many of them could not be used for the purpose of this work for some reason or another. One of the major reasons preventing their use was in most cases due to the fact that it was often extremely difficult to determine with a reasonable accuracy the precise instant when the oncoming vehicles reached a reference line drawn on the screen for the purpose of calculating the headways. Most of the films having indeed been taken at intersections for specific ramp studies, our reference line was often located so far in the field of vision as to make accurate determination of the instant that a given part of any particular vehicle crossed the line practically impossible. Another reason which prevented their use was that on many films, the field of vision was so small that it did not show a point approximately halfway between the beginning of the full width of the deceleration lane and the island nose, which was the location where we referenced the vehicles. Fig. No. 4 is a sketch showing the general location of the reference line. Ideally, we would have liked to reference the vehicles before they reach the taper section of the deceleration lane, but no film provided us with this opportunity. In order that our data reflect the existing conditions of the traffic flow, we had to locate our reference line before the nose of each intersection so that we take an account of the vehicles using the deceleration lane; we had to reference them also at a point far enough from the beginning of the deceleration lane so that the through traffic previously influenced by the vehicles using the deceleration lane would be given enough time to readjust their operation as if they had not been influenced by the former.

After examination of all the films that were available, four films were finally selected for longitudinal movement analysis. All films show major intersections with the Highway #401, an urban freeway in the northern part of Metropolitan Toronto. Figure No. 5 is a sketch showing the study locations. To simplify the naming of the sites throughout the rest of the thesis they will be called only by the name of the street which intersects Highway #401.



SKETCH SHOWING THE GENERAL LOCATION
OF THE REFERENCE LINE

FIGURE 4



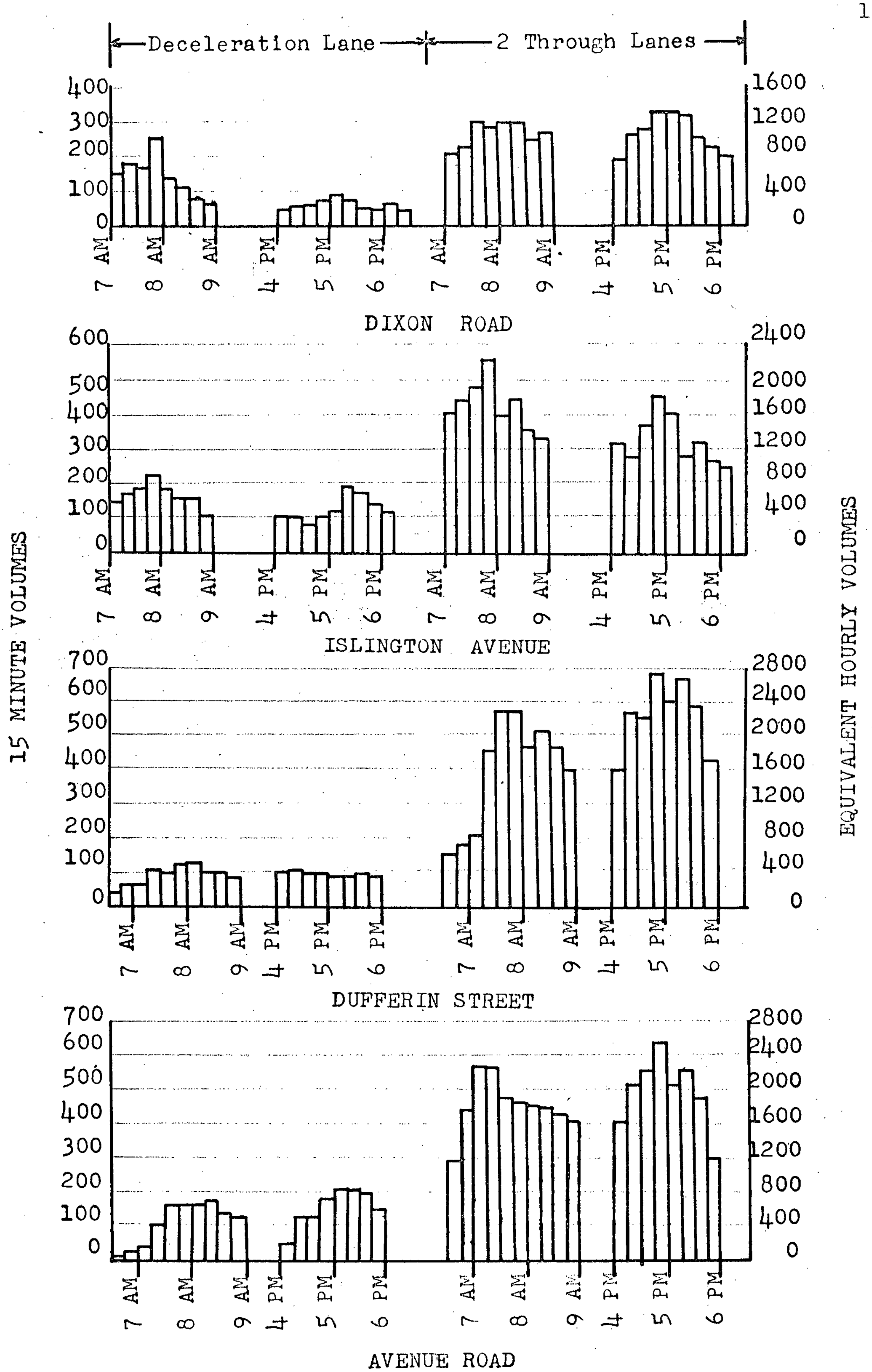
Sketch Showing
The Study Locations

FIGURE 5

While each intersection has a different layout, this was not taken into consideration in our study since the longitudinal movement of vehicles which we study is not influenced by the intersection layout. Each intersection is in flat terrain and the geometric features of both the interchanges and the highway at each location have the same standard level of design so that neither the geometric design of the highway nor the physical nature of the highway have any effect on the spacing of vehicles at any particular location.

The Annual Average Daily Traffic volume at these locations as determined by the Ontario Department of Highways was reported as being between 36,000 and 67,000 vehicles per day for both directions of travel. Some typical traffic volumes taken at each location with portable automatic traffic recorders are shown in Fig. No. 6; these volumes have been recorded during one week just prior to the filming. The highway was originally designed as a rural freeway, but due to its proximity to Toronto and the large volumes of traffic, it can be classed more accurately as an urban freeway.

All the films used in this study were taken during that period of time when there was the greatest combination of through and diverging traffic. This happened from 7:30 a.m. to 8:30 a.m. for westbound traffic



FIGU.E 6: TYPICAL TRAFFIC VOLUMES OBTAINED FROM AUTOMATIC TRAFFIC RECORDERS on Highway #401 and from 4:30 p.m. to 5:30 p.m. for eastbound traffic. Although it might have been preferable that the films be taken at different periods of the day in order to deal with the greatest range of traffic volumes, we feel that the volumes obtained for each individual lane at each intersection provided us with a range of volumes good enough for the purpose of this thesis. Table No. 1 shows these hourly volumes projected in each case from our population samples. In other words, we think that the volumes of traffic which were analysed show enough variations so as to serve our purpose of studying the longitudinal flow pattern of traffic at different volumes.

EXTRACTION OF DATA

Our work consisted of observing at each location the headways in individual lanes and in the two combined through lanes. Although the method used was very tedious and required a considerable period of time, it was very simple. It consisted of turning the projector manually until the right front tire of a vehicle was approximately even with our screen reference line and then of recording the corresponding frame number. Knowing the camera speed, which in our case was always 8 frames per second, the headways could be simply obtained by subtracting the consecutive counter readings and then dividing by the

TABLE NO. 1

SUMMARY OF THE PROJECTED HOURLY VOLUMES (IN VPH)

LOCATION	DRIVING LANE	PASSING LANE	DECELERATION LANE	TOTAL ALL LANES
DIXON RD.	460	658	846	1964
ISLINGTON AVE.	562	1096	674	2332
DUFFERIN ST.	950	1614	328	2892
AVENUE RD.	852	1788	622	3262

camera speed, which was 8 frames per second. We followed the same method for the four study locations. At each location, we ran the film three times, first recording simultaneously the headways of both the driving and passing through lanes, secondly recording the deceleration lane headways and thirdly recording the headways for the two combined through lanes.

At first, we recorded the headways for periods of fifteen minutes, which was approximately half of the film spools, and when headways for each lane had been recorded, we automatically reran the film in its entire length, which corresponded approximately to a period of time of thirty (30) minutes, this time counting the vehicles in each lane. The next step was to project these volumes to hourly volumes by assuming a linear relation which we felt would be accurate enough since the filming having been done at peak hours, the volumes of traffic observed were so heavy and the traffic flow so regular as to assume the same regularity for the second half hour. This has proved to be true at a later stage of this thesis when we decided to reduce our samples to 200 vehicles to simplify the calculations and we found negligible differences between the volumes projected from the time corresponding to the first 200 headways and those projected from the volumes observed during the entire length of the spools. The headway sample means calculated from both sets of data also showed negligible differences. Therefore, we have good reasons to affirm that the estimated hourly volumes projected from our sample means closely represent the existing hourly volumes at the time of filming. In any future similar type of study, we would recommend that traffic counts be taken at the time of filming so as to give an opportunity to check the estimated figures.

III. EXPERIMENTAL RESULTS

THE CONCEPT OF HEADWAY

The longitudinal pattern of traffic flow can best be analysed by the measurement and examination of the gaps between vehicles. But this can be done in several ways. One way is to measure the distance in feet between vehicles. Another way is to measure the distance from the beginning of one vehicle to the beginning of the next vehicle. In either case, individual or average spacings between vehicles can be determined. However, from observation, it has been found that the minimum or "desirable" spacing (i.e. the distance to the vehicle ahead that the motorist accepts as a safe distance so that he will have enough time to react in case of emergency) varies as some function of the velocity at which the vehicles are travelling. Thus while a distance of 50 feet to the preceding vehicle may be acceptable at a speed of twenty five miles per hour, the motorist would no doubt prefer a greater distance if his speed is sixty miles per hour. Therefore we can see that it is not sufficientto compare the spacings of vehicles in terms of linear distances without considering also the vehicle speeds.

A more convenient method to measure these gaps is in terms of units of time rather than distance, the second being the usual unit used. The elapsed time from the passing of the front (or any other part) of a vehicle past a fixed point until the same part of the next vehicle passes the same point is termed a "headway". The examination of headways in a traffic stream does not have to be correlated with speeds since a desirable headway does not vary much with speed. The variation of desirable headway as related to variation of speed is very small and can be neglected. Another reason why the "headway" concept has to be preferred to a linear spacing concept in stream analysis is that while the latter is a direct measure of traffic density, the headway is a direct measure of the rate of traffic flow, which is easier to determine and at a lower cost than the traffic density. Therefore, this concept has been used throughout the length of this thesis.

A study conducted by O. K. Norman, of which the main results have been summarized in Figure 7-6 of Traffic Engineering by Matson, Smith and Hurd(7) suggests that a headway greater than nine (9) seconds is considered to mean that the following vehicle is operating in a free flow condition and is not influenced by the vehicle ahead. For headways smaller than nine (9) seconds, the absolute speed of the rear vehicle begins to fall rapidly to approach the speed of the vehicle ahead, and there is a marked drop in the relative speed of the first and trailing vehicles, thus indicating that the trailing vehicles adjust their speed to the leading vehicle, and therefore that the former is operating under restricted flow conditions. The preceding vehicle is considered to exert some influence over the behaviour of the following vehicle.

Although we expected to observe in our study a great number of headways smaller than nine (9) seconds, thus losing that "complete randomness" upon which all the theory of probability and statistics is based, we decided that a statistical approach was justified since we felt that there still was left in our traffic stream a certain degree of undetermination and randomness. Furthermore, this was our main objective to find out where a statistical approach could be applied or at what point such an approach ceased to be realistic.

THE POISSON FORMULA

Previous observations have shown that the distribution of headways in a traffic stream does not display a central tendency about the mean, i.e. is not a "normal" distribution in a statistical sense. The experience has shown that most of the drivers tend to travel at headways less than average while only a few drivers exceed the average value, usually by a much larger amount. Fig. No. 7 is a typical headway distribution curve. This suggests that the headway distribution might follow the function derived from probability laws known as the Poisson distribution. If we examine our own data for one of each type of lanes studied (see Fig. 8, 9, 10, 11), it becomes readily apparent that the resulting curves have approximately the same shape as the previous typical headway distribution. It seemed therefore reasonable to analyse in more detail our experimental observations, even if it was known that theoretical and observed results would not coincide exactly.

The Poisson distribution is given by:

$$P(x) = \frac{e^{-m} m^{x}}{x!} \qquad (1)$$

where $P(x)$ = the probability of x events occurring. $x = 1,2,.....n.$

m = the expected number of events occurring on any given observation, i.e. the mean of x, ave (x) or $\overline{X} = m$.

e = the base of Naperian logarithms = 2.71828.

Applied to traffic, the definition of terms becomes:

$P(X)$ = probability of the arrival of x vehicles at a point during a given length of time.

m = mean number of vehicles arriving in the given length of time = $\frac{tV}{3600}$.

t = given time length of gap (sec.)

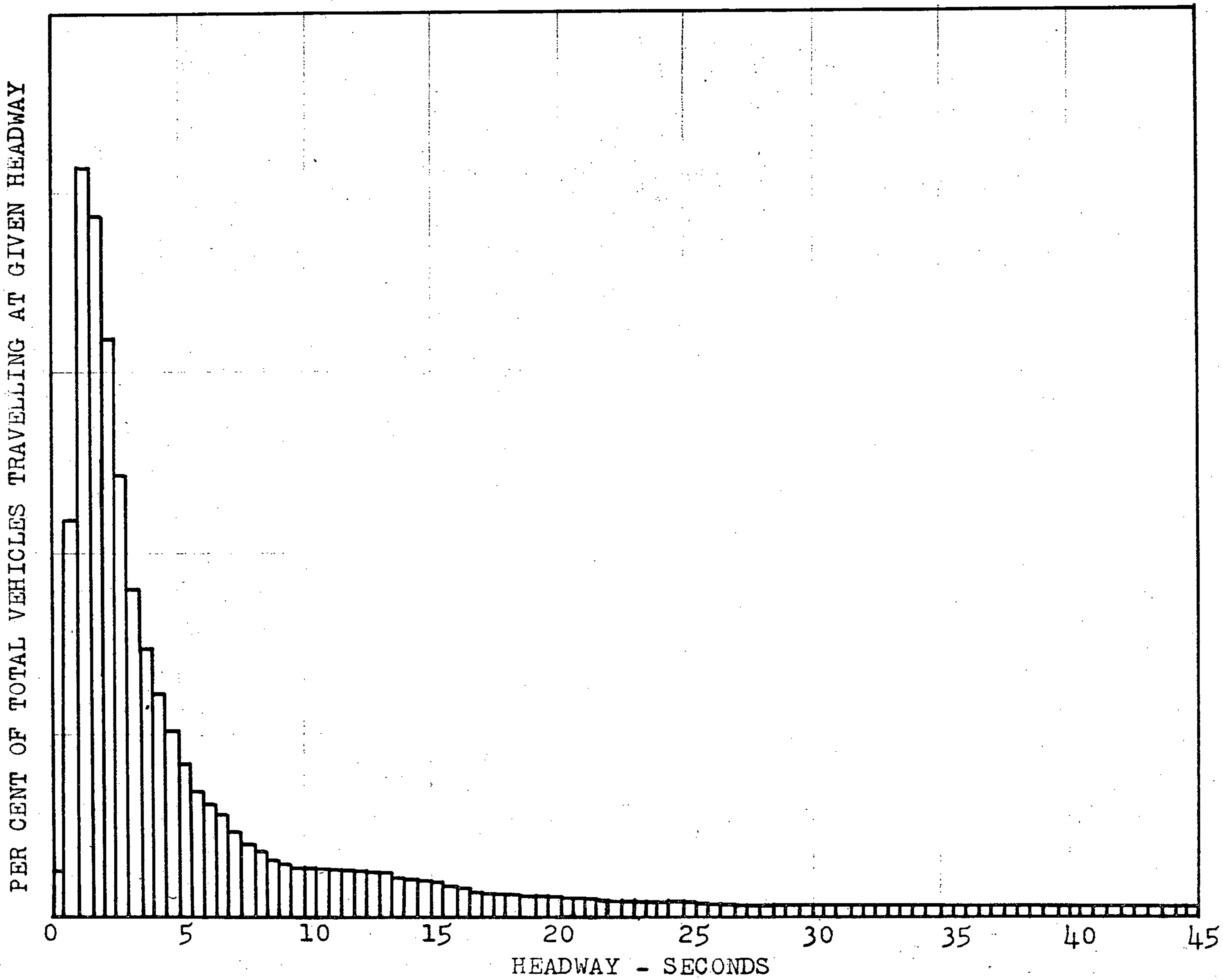


FIGURE 7: A TYPICAL HEADWAY DISTRIBUTION CURVE

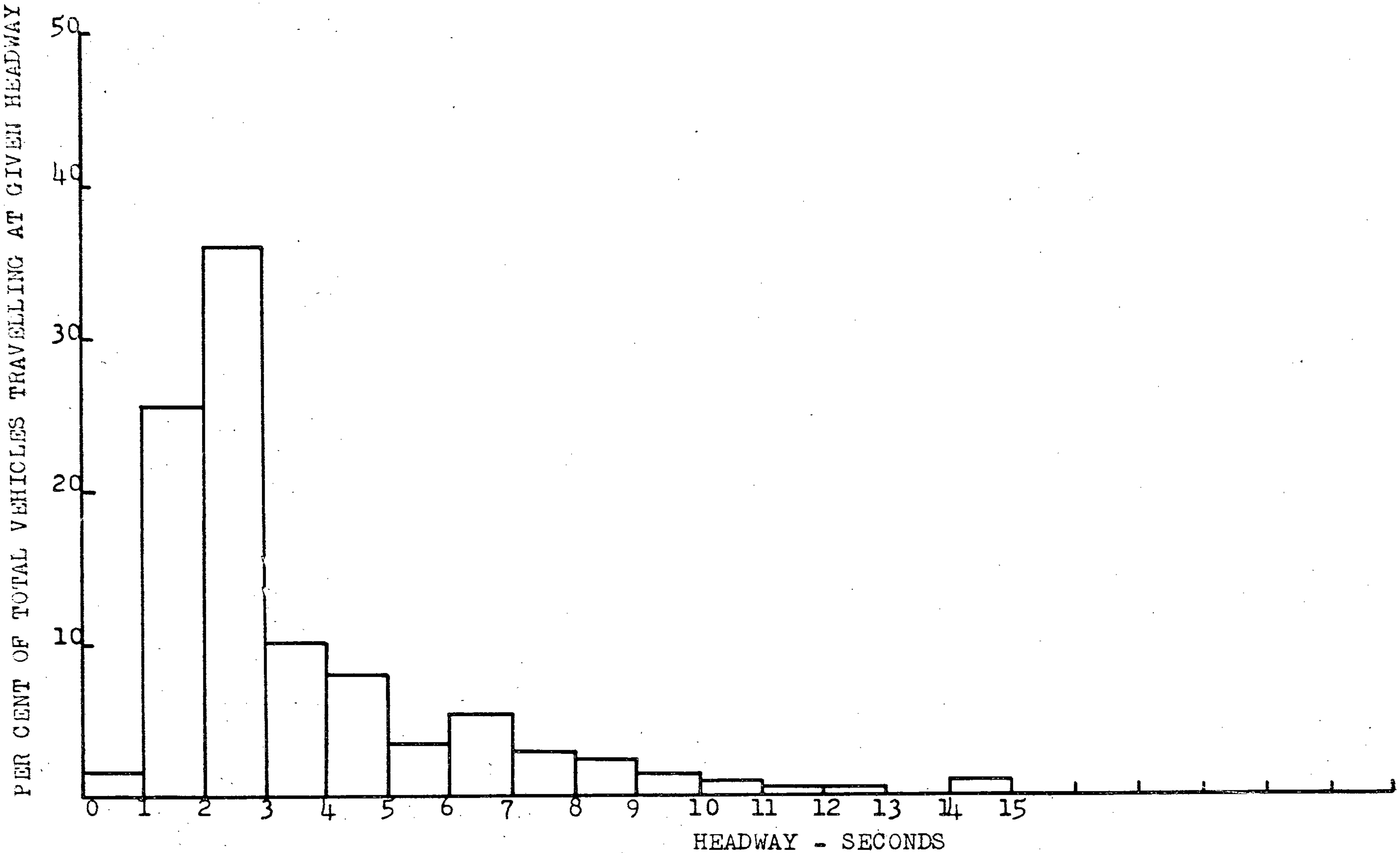


FIGURE 8: OBSERVED DISTRIBUTION OF HEADWAYS IN A DRIVING LANE (DUFFERIN STREET)

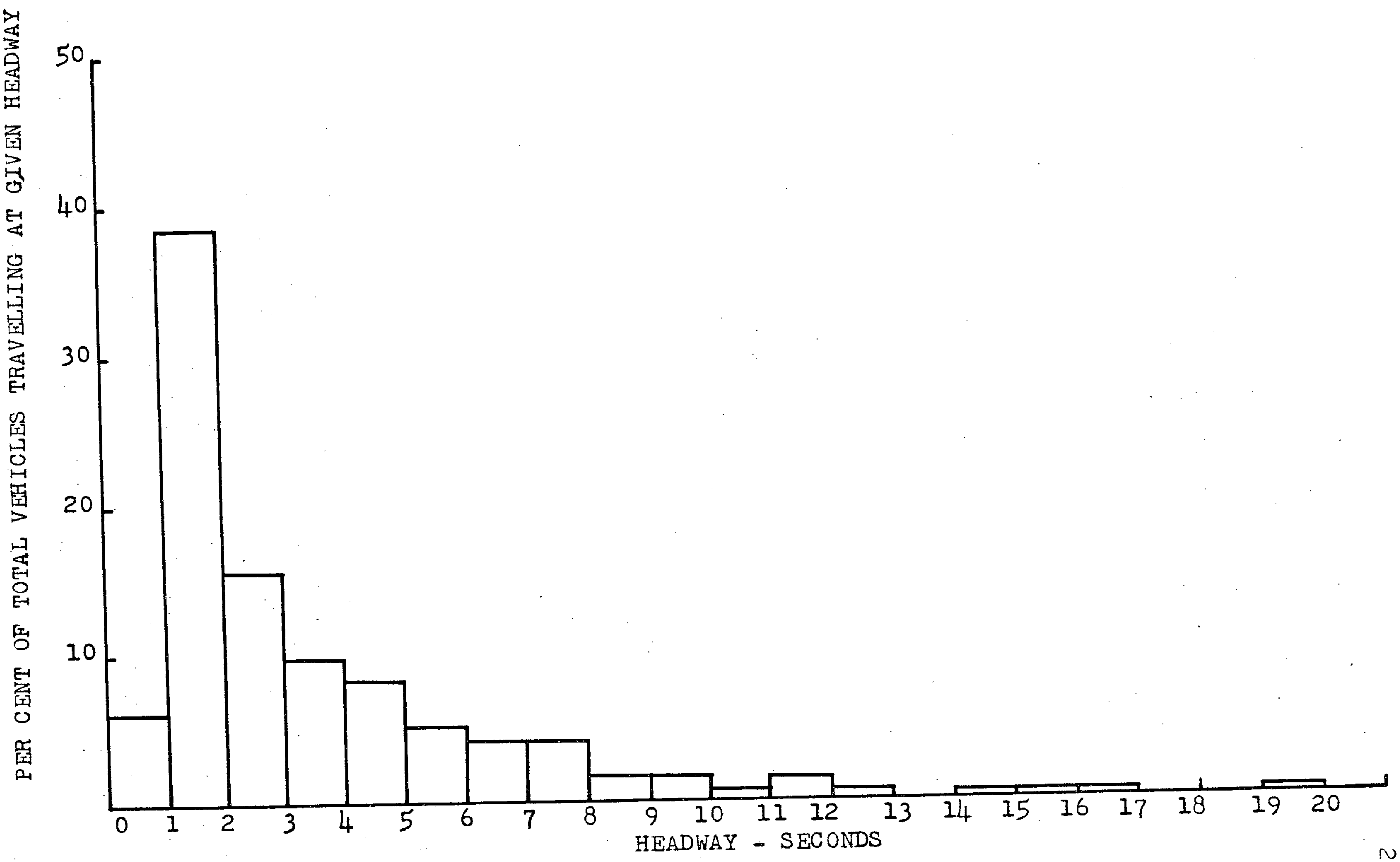


FIGURE 9: OBSERVED DISTRIBUTION OF HEADWAYS IN A PASSING LANE (ISLINGTON AVENUE)

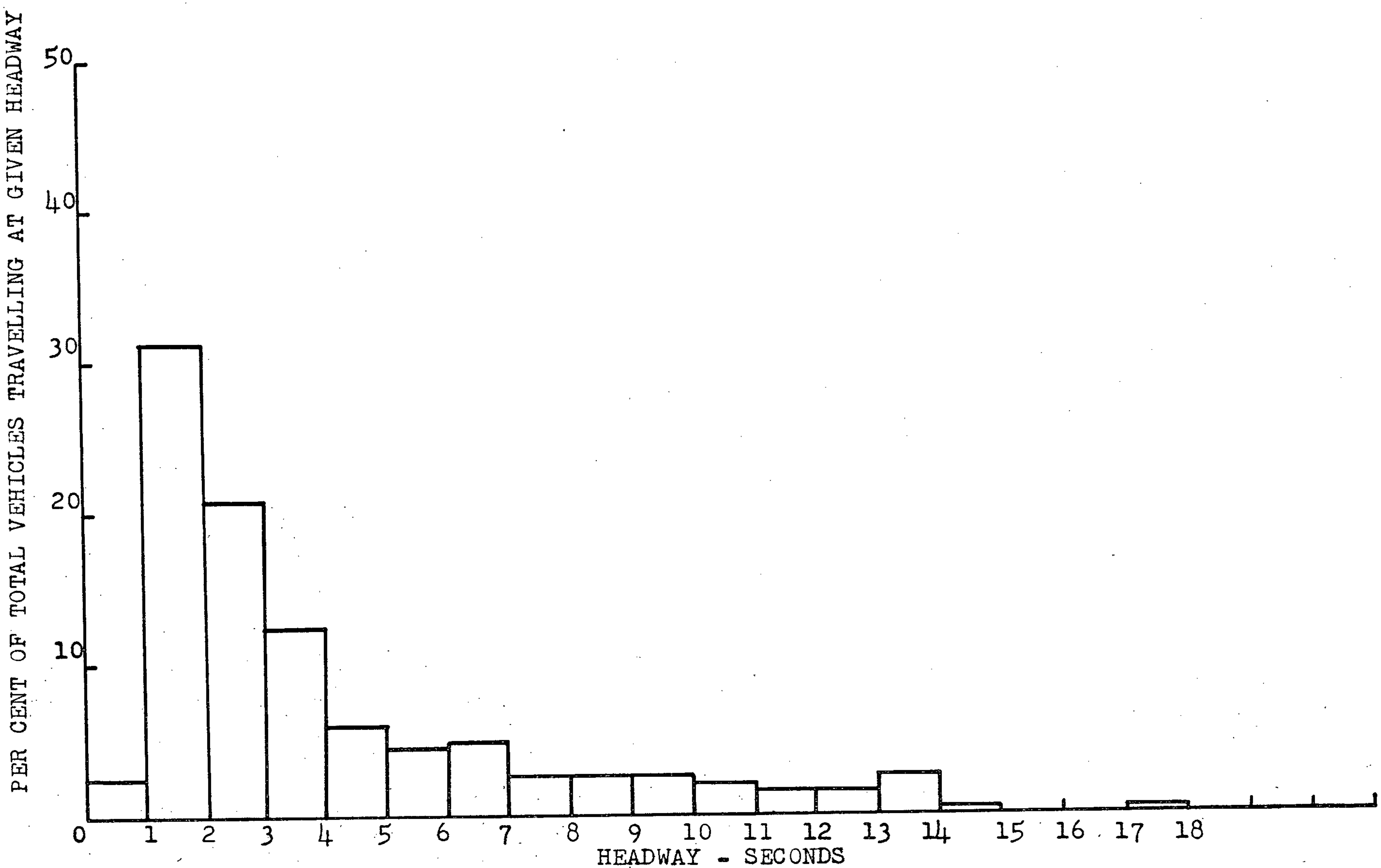


FIGURE 10: OBSERVED DISTRIBUTION OF HEADWAYS IN A DECELERATION LANE (DIXON ROAD)

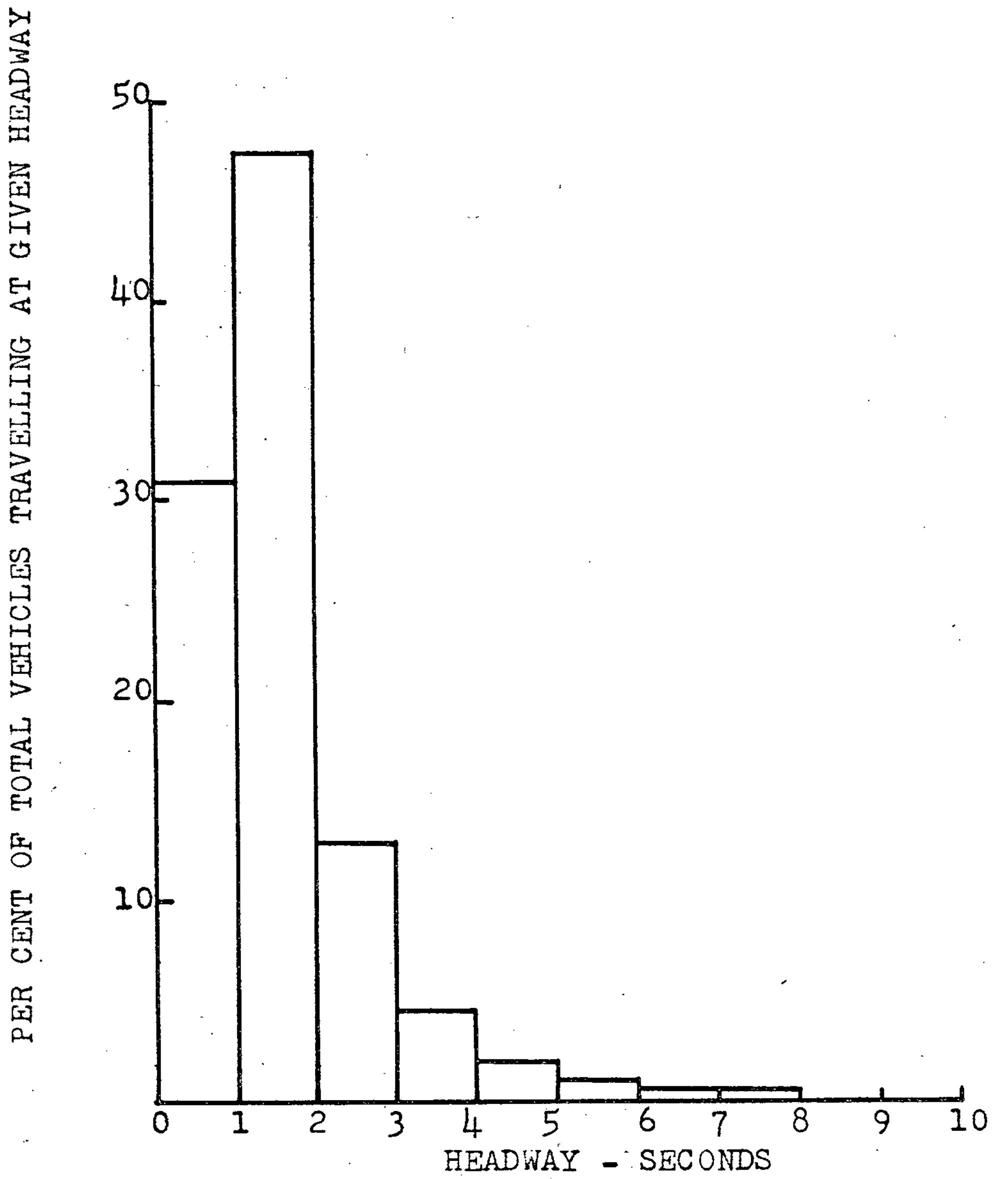


FIGURE 11: OBSERVED DISTRIBUTION OF HEADWAYS IN THROUGH LANES COMBINED (AVENUE ROAD)

V = hourly volume in VPH.

If the hourly volume is not known, $m = \frac{t}{\bar{x}}$, where $\bar{x}$ = the average of observed headways. Since a headway corresponds to a period of time during which no vehicle arrives, substituting zero (0) to x in the above formula (1) gives:

$$P(0) = \frac{e^{-m}m^{0}}{0!} = e^{-m} \qquad (2)$$

which may be interpreted as the probability of occurrence of headways equal to or greater than a selected time t. Following what was mentioned above, formula (2) can be written in two different ways, as follows:

$$P(t) = e^{-vt/3600} \qquad (3)$$

where P(t) = the probability of occurrence of a headway greater than t seconds

and V and t are defined as above;

$$\text{or} \quad P(t) = e^{-t/\bar{x}} \qquad (4)$$

where P(t), t, and $\bar{x}$ are defined as previously.

Since the hourly volumes were not known at the time of computations, we used formula (4) rather than formula (3) but we would have obtained the same results in using the latter since the hourly volumes have been based upon the headway sample means and since (as previously stated) the proportion between their time of observation and the hourly volume was a linear one.

SAMPLES

As previously mentioned, when we began extracting the data with the projector our samples were drawn from a film running time of approximately half a film spool or 15 minutes. The result was that our sample varied from lane to lane and from location to location, since we dealt with different volumes during the same elapsed period of time. At a later stage however we realized that using a constant sample would greatly simplify the calculations and make them less tedious. Therefore, instead of drawing our sample from a constant time of observation, we observed a constant number of cars, no matter what was the elapsed time of observation. We felt that a sample of 200 cars would be sufficient to obtain a statistically stable sample and therefore decided to analyse this sample at each location. This sample stability was well demonstrated since we obtained approximately the same hourly volumes and the same sample means when we calculated them using in all cases the two sets of samples.

SAMPLE MEANS AND VARIANCES

There were two ways to calculate the sample mean. One was to observe all headways of our samples, sum them up and then divide by the total number of observed headways, which in our case was 200. Since this method required that we knew the exact value of each headway, it was rejected since it involved a considerable amount of calculations if we consider that we had to study a total of sixteen (16) samples, each one involving two hundred (200) figures. Another one was to divide the total time of observation by the number of vehicles observed. This method, much simpler than the latter, was used. The total time of observation could be easily calculated by recording the frame number corresponding to the two hundredth (200^{TH}) vehicle and then dividing by the camera speed (8 frames per second). If we let Y = the frame number corresponding to the passage of the 200th car, the sample mean $\overline{x}$ is given by:

$$\overline{x} = \frac{\text{TOTAL TIME OF OBSERVATION}}{\text{TOTAL NUMBER OF OBSERVED VEHICLES}} = \frac{Y}{8 \times 200} = \frac{Y}{1600} .$$

The sample mean $\overline{x}$ being known for each set of data, the sample variance S^2 could easily be obtained by using the well known formula:

$$S^2 = \sum_{i=1}^{K} \frac{(x_i - \overline{x})^2 f_i}{n-1}$$

where S^2 = the sample variance

$\bar{x}$ = the sample mean

K = the number of cells

xi = the cell midpoints

fi = the cell frequencies

n = the total number of observations = $\sum_{i=1}^{K} fi = 200$

HOURLY VOLUMES

The hourly volumes projected from our samples have been obtained as follows:

$$\text{Hourly volume} = \frac{\text{No. of observed headways x camera speed x 3600}}{\text{Frame number corresponding to the 200th vehicle}} + 1$$

$$= \frac{200 \times 8 \times 3600}{Y} + 1$$

As we noted earlier, we have good reasons to believe that the theoretical hourly volumes calculated by the above formula coincide closely with the actual volumes which would have been recorded, had we counted the traffic volumes during one hour at the time of the filming. Table No. 2 gives a summary of the calculated sample means, sample variances and hourly volumes for each individual lane and through lanes combined.

OBSERVED HEADWAYS

Having a mathematical tool to work with, it is now possible to examine the data obtained in the present thesis

TABLE NO. 2

SUMMARY OF CALCULATED SAMPLE MEANS ($\bar{x}$), SAMPLE VARIANCES (S^2), HOURLY VOLUMES AND $1/\bar{x}$.

LOCATION	TYPE OF LANE	SAMPLE MEAN ($\bar{x}$) (SEC)	$\frac{1}{\bar{x}}$	SAMPLE VARIANCE (S^2)	HOURLY VOLUME (VHP)
DIXON ROAD	DRIVING	7.85	0.127	30.16	460
	PASSING	5.48	0.182	32.18	658
	DECELERATION	4.26	0.235	16.72	846
ISLINGTON AVE.	DRIVING	6.42	0.156	17.96	562
	PASSING	3.28	0.305	9.78	1096
	DECELERATION	5.35	0.187	28.53	674
DUFFERIN ST.	DRIVING	3.80	0.263	6.45	950
	PASSING	2.23	0.448	1.64	1614
	DECELERATION	10.97	0.091	102.32	328
AVENUE ROAD	DRIVING	4.23	0.236	8.97	852
	PASSING	2.01	0.498	1.88	1788
	DECELERATION	5.80	0.172	28.43	622
DIXON ROAD	2 THROUGH L.	3.22	0.311	7.45	1118
ISLINGTON AVE.	2 THROUGH L.	2.17	0.461	3.79	1658
DUFFERIN ST.	2 THROUGH L.	1.40	0.712	1.23	2564
AVENUE ROAD	2 THROUGH L.	1.36	0.733	1.29	2640

and see how close they fit with the theoretical Poisson distribution. In order to do this, we first calculated the theoretical Poisson curve for each set of data, using the formula $P(t) = e^{-t/\bar{x}}$ in which $\bar{x}$ had already been calculated (see Table No. 2). This was easily done with the help of Exponential Tables(9). The next step was to classify our observed headways in cells of one (1) second intervals and to tabulate the observed cumulative cell frequencies. Since the theoretical distribution is a 100%-cumulative frequency distribution (i.e. it gives the % of headways greater than a given value), we also tabulated the 100%-cumulative cell frequencies of the observed headways in order to plot and compare the theoretical curves with the one obtained from our data. Tables A-1 to A-16 (in Appendix A) and Fig. B-1 to B-16 (in Appendix B) show the observed results and their corresponding curves together with the theoretical results and their corresponding curves; also included are the theoretical results and curves obtained from the Erlang distribution, used at a later stage of this thesis.

When we observe the Tables and Figures mentioned above, we notice great discrepancies between the experimental and theoretical results. We certainly cannot conclude by a mere observation of the curves a very close fit between the observed headways and the theoretical probability curves. This wasto be expected because of the nature of the Poisson distribution (3). Being derived as a limiting form of the binomial distribution it applies only when the number of possible events is large (theoretically, infinite) and the probability of occurrence of any individual event in the time interval considered is small. An example of the kind of data which we would expect to find distributed in the Poisson form would be the number of accidents happening at a very busy intersection; or the frequency of headways of a given length on a road where the traffic volume is extremely small. This is obviously not the case at our study locations. The traffic volumes observed at the time of filming are so heavy that the probability of occurrence of a headway of any small time length is very large and the number of possible large headways is very small. The reason for studying the headways on the deceleration lanes was precisely that when we began our study, we thought that these lanes would carry the small volumes for which a Poisson distribution might apply. Unfortunately, after projection of our data to hourly traffic volumes, we discovered that only one out of the four (4) deceleration lanes studied had a relatively small volume. This happened in the deceleration lane at Dufferin St., where the projected hourly volume was 328 vehicles per hour. Of all the Figures, Fig. B-9 is probably the one where we can observe the closest fit between the experimental curve and the theoretical

Poisson curve. Although the two curves do not fit perfectly, they are very close to each other for headways greater than four (4) seconds. A statistical test performed at a later stage of this thesis to check the goodness of fit of the experimental and theoretical curves will show that even if the two curves above mentioned do not fit statistically, they fit much better than any other similar set of two curves at each study location. Although the statement of definite conclusions would have to be supplemented by other studies, we feel that this is an indication that the Poisson distribution may be the best model to describe the headway distribution at small volume of traffic. This also might be an indication of the volume which should be defined as a "small volume" in order that the Poisson function gives a fit.

From the observation of the Figures, we notice that all curves follow approximately the same general pattern. But there are so many variations among them that we can hardly find a relation between the traffic rate of flow and the goodness of fit of the observed headway distributions with the theoretical Poisson distribution. If any trend is to be observed, it might be that in most cases (if not in all) the discrepancy between the theoretical and experimental curves tends to decrease as the headway time length "t" increases. This suggests that the observed data might be represented better by two distributions, one for small spacings and one for large spacings. This idea of a "double distribution" was first suggested by Greenshields (2) who once obtained a good fit with a distribution for headways less than 4 seconds and another for headways of more than 4 seconds. It would therefore be interesting to check whether our data follow this kind of pattern.

As already mentioned, the Poisson function applied to the distribution of headways is of the general form $y = e^x$ (see eg. 2) which may be written:

$$\log_e y = x.$$

Thus the equation plotted on semilog-paper becomes a straight line with a negative slope since $x = -m$. When we plot our data on semilog-paper, we get the curves shown on the Figures C-1 to C-16 in Appendix C. It appears that the greatest discrepancies occur for large headways but this is not the case and simply due to the semi-log scale. In only 3 cases out of 16 the data were more closely fitted by two straight lines. And of these three, it will be seen later that one fits the Poisson distribution at the 2.5% level and therefore should not give a better fit with two straight lines. The fact that the combined through lanes of Dixon Road (see Figure C-13) appear to be best represented by two lines when in fact it fits the Poisson formula is here again due to the semi-log scale. Indeed, we had to break the line for only a few points which are found in the larger headway range where we find the minimum deviations between the experimental and theoretical results but at the same time the maximum discrepancies due to the semi-log scale. In the Islington driving lane (Fig. C-4) and deceleration lane (Fig. C-6), the only cases left where we have the two straight lines pattern, the change of direction on the lines occurs at approximately 13 and 9 seconds respectively. Although the intersection of the two straight lines is very sensitive with respect to position of the two lines and these lines are only band fitted within a specified range, we do not believe that this is enough to explain the discrepancy in the "location of the breaks" of the curves which in Greenshields' study occurred at 4 seconds. From all this, it appears that the theory developed by Greenshields can hardly be generalized but is rather determined and influenced in each case by different location factors. Other studies previously done on the same subject by J. L. Vardon (11) and J. W. Wise (13) also showed great discrepancies with Greenshields' curves and therefore prove that we cannot generalize this concept of a multiple distribution, each random in its limited case, as one which would apply in all cases.

Greenshields (2) has also shown that for each plotted point there is a corresponding range of expected error or natural uncertainty caused by the fact that "unless the sample is very large there is always a difference between the sample values and those of the universe". This natural uncertainty, based on the standard deviation of the sample, is obtained from the formula:

$$Z = \sqrt{\frac{n}{n-1} fo \left(1 - \frac{fo}{n}\right)} \qquad (5)$$

where n = the total number of happenings recorded

fo = the accumulated frequency.

Since in our study n was always equal to 200, the factor $\frac{n}{n-1}$ is so very close to unity that it can be neglected and the equation becomes:

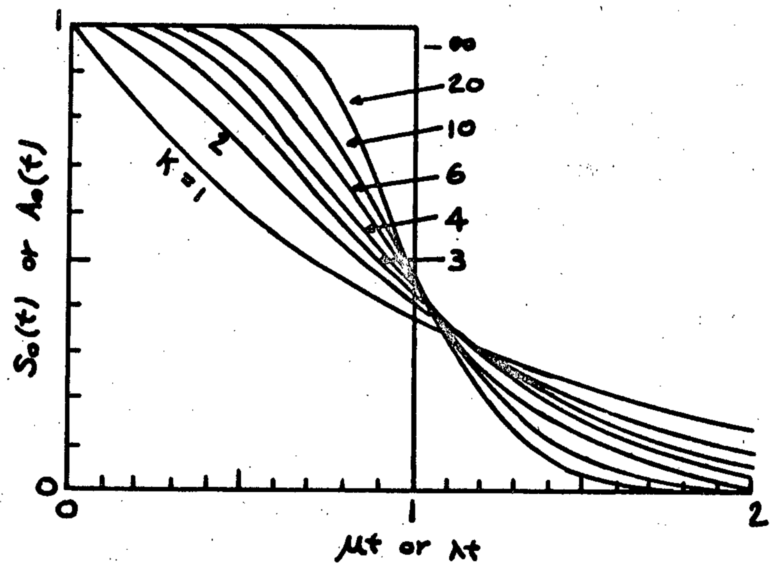
$$Z = \sqrt{fo \left(1 - \frac{fo}{n}\right)}$$

These figures have been calculated for each point and plotted in Figures C-1 to C-16. With a few exceptions for points in the (0-2) second interval, the lines drawn through the experimental points stay within the natural uncertainty range, from which we can conclude that the data can be represented by a straight line. Even in the combined through lanes of Dixon Road and the driving and deceleration lanes of Islington Avenue, where the data were fitted by two straight lines, a single line would stay very closely within the expected range of error.

Although the best fitting curve appeared to be in most cases a single straight line, it is easily observable from the Figures C-1 to C-16 that (with the exception of Fig. C-9) the distribution is not a Poisson one. Everywhere, especially in the small headway range, we observe great variations between the two distributions and the slope of the experimental and theoretical curves are different. The fact however that the experimental data can be closely fitted on a semi-log paper by a straight line means that the model best representing them still has some exponential form, more elaborate than the Poisson model and with a different slope.

THE ERLANG DISTRIBUTION

From Tables A-1 to A-16 and Figures B-1 to B-16, we observe many discrepancies between the experimental distribution and the Poisson distribution. But a general pattern common to all cases is that in the smaller headway range the experimental curve lies somewhere well above the Poisson curve. Thus the model that we need to represent our data is one which would give greater probability values in this range of small headways. We felt that the best distribution serving this purpose is the A. K. Erlang distribution. Figure 12 shows the Erlang distribution



ERLANG ARRIVAL or SERVICE-TIME DISTRIBUTION.
Probability that the next arrival will occur after time interval t.

FIGURE 12

for several values of the parameter K; where K=1, the distribution is the simple exponential or Poisson case. The Erlang distribution has been extensively used in telephone traffic for different purposes such as to calculate the line holding or waiting time and we felt that similarities between telephone and highway traffic in crowded situations are such that it might well be applicable to highway traffic.

NATURE OF THE ERLANG DISTRIBUTION

The Erlang distribution can be thought upon as if every oncoming vehicle had to pass through a series of exponential channels, called phases, be held in the exponential channel for a variable time and then released from the channel before the following vehicle can be accommodated. Each phase is of the exponential or Poisson type but the resulting distribution is not exponential. There is a possibility of forming any kind of distribution pattern by simply adding more and more phases. These distributions, called Erlang distributions, provide a family of service-time distributions which range from the completely random exponential type to the completely regular service-time situation. This kind of service-time distribution can be easily interpreted as applying to a freeway toll booth operation. In such a case, it is perfectly imaginable that a vehicle has to go through any number of phases that we want before being accommodated by the toll booth facility and the following vehicle being allowed in the same booth. This is not so easily interpretable when applied to the headway distribution. It is indeed hardly imaginable that the traffic flow actually goes through such phases. Or if it does so, it is still more difficult to physically separate its operation into distinct phases. The only physical interpretation that we can think of would be to assume a traffic stream composed of two types of drivers, each type representing a different phase. One phase would include all those drivers who are always in a hurry and therefore always keep the distance between their vehicle and the leading vehicle (the headway) to a minimum. As soon as they can find in the parallel traffic lane a minimum acceptable gap, they shift lanes and overtake the leading vehicle. The other phase would be composed of all the drivers who, not being in a hurry, don't mind to have any gap length between their vehicle and the leading vehicle. These drivers are assumed to operate their vehicles as if they were not influenced at all by the operation of the other vehicles in the traffic stream. Although this interpretation may not be totally in conformity with the nature of the Erlang service-time distribution types, it is the best that we could think of. On the other hand, it was not so important to us to visualize a physical interpretation to a mathematical model as to find a mathematical model which would best fit our experimental data. The Erlang distribution being "less random" than the Poisson distribution since it predicts fewer very short or very long intervals between arrivals, it appeared to be a mathematical model that might give a better fit to our observed headway distribution than Poisson.

The Erlang distribution is given by the formula:

$$A_o(t) = e^{-K\lambda t} \sum_{n=0}^{K=1} (K\lambda t)^n / n! \qquad (6)$$

where:

$A_o(t)$ = the probability that no arrival occurs in time t after the previous one.

λ = the mean rate of arrival.

K = the number of phases in the system.

n = the number of states in the system.

e = the base of Naperian Logarithms = 2.71828.

Applied to traffic, the equation (6) becomes:

$$P(t) = e^{-K\lambda t} \sum_{n=o}^{K=1} (K\lambda t)^n / n! \qquad (7)$$

where,

P(t) = the probability of occurrence of a headway greater than t seconds.

$\lambda = 1/\bar{x}$, $\bar{x}$ being the average of observed headways.

Substituting K=1 in (7) gives:

$P(t) = e^{-t/\bar{x}}$, which is the familiar exponential or Poisson distribution (see eq. 4).

After a careful examination of the family of Erlang curves as shown in Figure 12, we felt that the Erlang distribution with two phases (or K=2) had the best chances to fit our experimental data. In order to check this, we tried to fit the experimental data of the Avenue Road driving lane with a three phase Erlang distribution (K=3); a test of goodness of fit proved that an Erlang curve with K=2 gives a better or closer fit. Then we decided to analyse the fit obtained from the latter at all study locations. Substituting K=2 in eq.(7) gives:

$$P(t) = (1 + 2\lambda t)\, e^{-2\lambda t} \qquad (8)$$

As we had previously done for the Poisson distribution, by substituting different values to "t" in eq. (8), we obtained for all sets of data the theoretical Erlang distribution with K=2. These results and curves have been summarized in Tables A-1 to A-16 and Figures B-1 to B-16.

In all cases, as we expected, this distribution gives a much closer fit than the Poisson distribution in the smaller headway range. Then follows a transition range of headways in which we observe great discrepancies between the experimental distribution and either one of

Poisson or Erlang. In this range, we don't observe either any regularity: in some cases, Poisson gives a better fit, in other cases it is Erlang. In the larger headway range, the discrepancies between the experimental data and the theoretical Poisson or Erlang curves decrease appreciably but here again we don't observe any regularity. As for the intermediate range, the experimental distribution sometimes lies between the Poisson and Erlang distributions, sometimes getting closer to either one of them but remaining inside; sometimes it goes outside either one of them. In general however, the observed distribution tends to be closer to the Erlang distribution.

From observation of the results and curves, it does not appear that our observed data can be represented by either one of the theoretical distributions. Furthermore, if one of them is to give a fit, we can hardly determine which one it would be since, although the Erlang distribution generally lies closer to the observed data, the determination of a goodness of fit based on a simple examination of curves is usually very misleading.

TEST OF GOODNESS OF FIT

Several statistical tests of significance exist which allow us to determine with more certainty if the experimental and theoretical data coincide. The chi-square

(X^2) test is the most appropriate to the present application.

By definition, $$X^2 = \sum_{i=1}^{K} \frac{(f_o - f_t)^2}{f_t} \qquad (9)$$

where,

f_o = the observed frequency for any class interval.

f_t = the computed or theoretical frequency for the same class interval.

K = the number of class intervals.

If n = the total number of observations, then $f_t = n\,p_i$, p_i being the probabilities associated with the class intervals. When we compare the statistics obtained from eq. (9) with the X^2 distribution with degrees of freedom $V=K-\gamma$, where γ is the number of linear restrictions imposed on the difference (f_o-f_t), we can test if the experimental data can be represented by any given theoretical distribution. The X^2 tabulated values are the maximum values which the sample statistic can assume and yet still be considered as representing that the experimental curve does not differ significantly from the theoretical curve. In other words, if the sample statistic obtained from eq. (9) is smaller than the tabulated value at a given significance level α, there is no evidence to indicate that the experimental distribution differs from the theoretical distribution. On the other hand, if the sample statistic exceeds the tabulated value at the same significance level α, we have $(1-\alpha)\%$ of chances of being right when we affirm that the experimental distribution differs from the theoretical data. We could also say that the probability is less than α % that the experimental and theoretical curves are the same. Common significance levels are 0.01, 0.025 and 0.05.

We can now use this test to check which one of either Poisson or Erlang distribution represents more closely our experimental data. For this purpose, we compared the experimental observed frequencies (f_o) with the theoretical expected class frequencies (f_t) derived from both distributions. As previously stated, the expected class frequencies are obtained by multiplying the number of observed frequencies (n) by the probability (p_i) associated with each class interval. Although these probabilities could have been obtained in tables in the case of the Poisson distribution, the fact that there were not such tables for the Erlang distribution lead us to decide not to use the Poisson tables but rather to use the same procedure for both distributions and calculate them directly from our results summarized in Tables A-1 to A-16. These tables give the probability of occurrence of a headway greater than a given time t, both for Poisson and Erlang distributions. The class interval probabilities (p_i) which were needed for the chi-square tests could simply be obtained by subtracting successively these tabulated values. Chi-square tests of significance have been performed for all study locations, each time comparing the observed data with both Poisson and Erlang distributions. These are shown in Tables D-1 to D-32 of Appendix D. The fact that the class interval probabilities (p_i) indicated in the above Tables have four (4) figures while the probability values of Tables A-1 to A-16 from which they have been derived have only three (3) figures is simply due to rounding of the latter figures. The chi-square test results have been summarized in Table 3, from which we can make a few observations. Firstly, only three (3) out of sixteen (16) sets of experimental data gave a statistical fit. At Dufferin deceleration lane and Dixon driving lane, we get a fit with Erlang distribution at the 1% level and a fit with Poisson at the 2.5% level in the combined through lanes of Dixon Road. The traffic volumes at these locations were respectively 328, 460 and 1118 VPH. This is totally opposite to our expectations, since we always felt that the Poisson distribution had best chances to give a fit for small traffic volumes and Erlang for heavy volumes. Secondly, out of the thirteen (13) study locations remaining, most of them (11) gave a closer fit with the Erlang distribution, with only two (2) being fitted more closely by the Poisson distribution. Thirdly, the combined through lanes, with the exception of Dixon Road which gave a fit with Poisson, all fit very nearly with Erlang.

From all this we can hardly suspect the existence

TABLE NO. 3

SUMMARY OF CHI-SQUARE TEST RESULTS

LOCATION	TYPE OF LANE	VOLUME	STATISTICS (POISSON)	CHI-SQUARE VALUE (1%)	DIFFER-ENCE	STATISTICS (ERLANG)	CHI-SQUARE VALUE (1%)	DIFFER-ENCE	CLOSER FIT TO
DUFFERIN	DECEL.	328	30.59	27.69	2.90	23.30	27.69	4.39	ERLANG*
DIXON	DRIVING	460	44.77	27.69	17.08	25.69	29.14	3.45	ERLANG*
ISLINGTON	DRIVING	562	65.39	24.73	40.66	26.74	24.73	2.01	ERLANG
AVENUE RD.	DECEL.	622	76.50	26.22	50.28	80.51	26.22	54.29	POISSON
DIXON	PASSING	658	47.13	26.22	20.91	87.03	26.22	60.81	POISSON
ISLINGTON	DECEL.	674	71.61	24.73	46.88	57.41	24.73	32.68	ERLANG
DIXON	DECEL.	846	76.60	23.21	53.39	60.84	21.67	39.17	ERLANG
AVENUE RD.	DRIVING	852	69.21	21.67	47.54	29.80	21.67	8.13	ERLANG
DUFFERIN	DRIVING	950	127.39	20.09	107.30	60.16	20.09	40.07	ERLANG
ISLINGTON	PASSING	1096	71.81	20.09	51.72	46.06	18.48	27.58	ERLANG
DUFFERIN	PASSING	1614	115.69	16.81	98.88	36.95	15.09	21.86	ERLANG
AVENUE RD.	PASSING	1788	163.70	15.09	148.61	71.32	13.28	58.04+	ERLANG
DIXON	2 LANES	1118	14.95	16.01	1.06	53.52	18.48	35.04	POISSONx
ISLINGTON	2 LANES	1658	31.12	16.81	14.31	20.06	18.48	1.58	ERLANG
DUFFERIN	2 LANES	2640	37.25	13.28	23.97	11.04	9.21	1.83	ERLANG
AVENUE RD.	2 LANES	2564	59.24	13.28	45.96	11.17	9.21	1.96	ERLANG

NOTE: x - Fits at the 2.5% level.
* - Fits at the 1% level.

of any relationship between the traffic volumes and the application of either Poisson or Erlang. It appears however that the Erlang distribution generally represents the headway distribution better than the Poisson distribution, although neither one gives an accurate representation of the actual observations.

SIMULATION BY PARALLEL ARRANGEMENT

The next step was to try another probability distribution which would fit the experimental data closer than the previous distributions. The Erlang distribution was defined for integer values of the parameter K and greater than unity. We feltthat it would be interesting to try a distribution where the value of K is smaller than unity. Such a value would correspond to a situation "beyond" Poisson, which might be called hyper-random. We felt fully justified in trying this new approach since the authors of a recently published research project (4) on the same subject declared: "there seems to be some evidence that a multi-lane freeway with very high traffic volumes might be hyper-random".

A model which gives such hyper-exponential curves is one, as in the case of the Erlang distribution, that assumes exponential channels. But unlike Erlang, where these channels were arranged in series, this one would assume a parallel arrangement of channels (8). It is represented by the density function:

$$So(t) = \sigma e^{-2\sigma nt} + (1-\sigma)\ e^{-2(1-\sigma)nt} \qquad (10)$$

When applied to traffic, eq. (10) becomes:

$$P(t) = \sigma e^{\frac{-2\sigma t}{\bar{x}}} + (1+\sigma)\ e^{-2(1-\sigma)\ t/\bar{x}} \qquad (11)$$

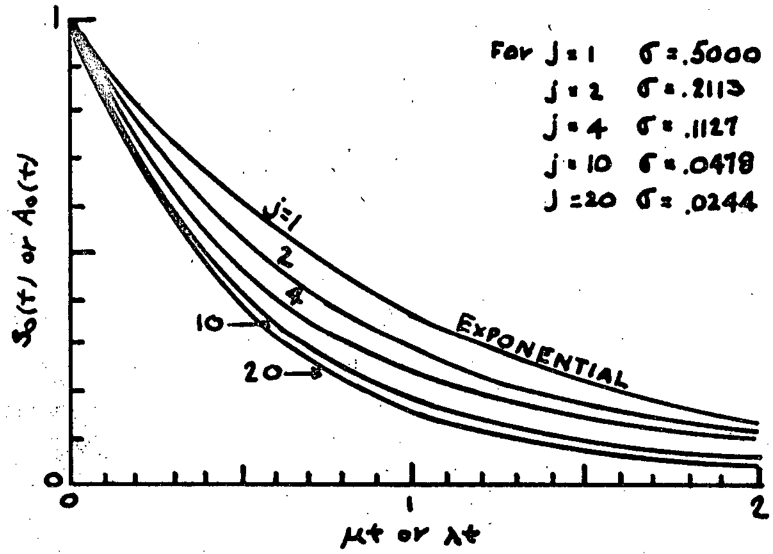
where P(t) and $\bar{x}$ have been defined in other sections of this thesis and $0 \angle \sigma \angle \frac{1}{2}$.

Our procedure was first to determine the parameter σ, which was given by the equation of the second degree:

$$s^2 = \bar{x}^2 \left[1 + \frac{(1-2\sigma)^2}{2\sigma(1-\sigma)}\right] = j\ \bar{x}^2 \qquad (12)$$

where j equals the expression in the brackets. ($j \geqslant 1$) and s^2 = the sample variance. Then, having determined σ, we could calculate P(t) from eq. (11), for different values of t. When looking at the curve patterns of eq.(10) which are shown in Figure 13, we had good reasons to believe that some of our set of data, especially when the observed distribution lies for a great part below the Poisson curve, would assume a similar type of distribution with different σ and j values.

When we solved eq.(12) by substituting the $\bar{x}$'s and s^2's values of all sets of data, we obtained imaginary roots for σ in all cases but one, and it was in the passing lane



FAMILY OF ARRIVAL DISTRIBUTIONS CORRESPONDING TO A PARALLEL ARRANGEMENT OF EXPONENTIAL CHANNELS.

FIGURE 13

of Dixon Road. The resulting equation was $4.07\sigma^2 - 4.07\sigma + 1 = 0$, whose solution gives $\sigma = 0.462$. Since $s^2 = j\,\bar{x}^2$, $32.18 = (5.48)^2\,j$, from which $j = 1.07$. Thus the theoretical distribution at the passing lane of Dixon Road would be an hyper-exponential one but located very close to the exponential or Poisson curve. This agrees with the results obtained when performing the chi-square tests. In this case we had found out a closer relation with Poisson than with Erlang but not a statistical fit. The only other case where Poisson gave a closer fit than Erlang was in the deceleration lane of Avenue Road. Logically, we would have expected here a fit with an hyper-exponential curve, although less than at Dixon Road since the discrepancies in the small headway range are larger. The fact that we did not obtain any real equation for σ in this case would indicate that the experimental distribution is better represented by a theoretical distribution located somewhere between Poisson and Erlang. We feel that the same conclusion could be applied to all cases where we did not obtain a fit with either Poisson or Erlang but where Erlang gave a closer fit. Although we did not compare our data with an Erlang distribution with K=3, we believe that the best fitting distribution is of the Erlang type with a K value between 1 and 2.

RELATION OF PARAMETER K TO VOLUME

Since the Erlang type of distribution appeared to us as a model susceptible to represent our experimental data, the next logical step was to find out if there is a linear correlation between the K values and the traffic volumes. If we could find such a correlation, we possibly could try to fit our data with a Gamma function which is an Erlang type distribution but has the advantage to work for any value of K. The coefficient of variation of K was $1/\sqrt{K}$ and was equal to the sample standard deviation divided by the sample mean. Thus

$$\frac{1}{\sqrt{K}} = \frac{s}{\bar{x}}$$, from which we get:

$$K = \frac{\bar{x}^2}{s^2} \tag{13}$$

The K values were first calculated for all individual lanes and plotted against the traffic volumes. Immediately it was obvious that the large amount of scatter ruled out a perfect relationship. The data was then analysed statistically to see if there was any linear relationship at all. The square of the correlation coefficient for pairs of measurements was calculated since this value would be a measure of the linear dependence of one set of values on the other. The traffic volumes were designated as xi while corresponding values of K were designated as Y_i. 12 pairs of measurements were suitable for analysis.

Then the square of the correlation coefficient (r_{xy}^2) is given by:

$$r_{xy}^2 = \frac{[n\sum x_i y_i - \sum x_i \sum y_i]^2}{[n\sum(x_i^2) - (\sum x_i)^2][n\sum(y_i^2) - (\sum y_i)^2]} \qquad (14)$$

In the present case,

$n = 12$; $\sum x_i = 10{,}450$; $\sum y_i = 20.23$; $\sum x_i y_i = 19{,}323.78$
$\sum(x_i)^2 = 11{,}256{,}428$; $\sum(y_i)^2 = 39.28$; $(\sum x_i)^2 = 109{,}202{,}500$;
$(\sum y_i)^2 = 409.25$.

Substituting these values in eq. (14) gave $r^2{}_{xy} = 0.261$, therefore the K factor is only 26.1% linearly dependent upon the volume of traffic.

The above test was then repeated using values for the combined through lanes. In this case,

$n = 4$; $\sum x_i = 7980$; $\sum y_i = 5.65$; $\sum x_i y_i = 11461.90$;
$\sum(x_i)^2 = 17{,}542{,}584$; $\sum(y_i)^2 = 8.04$; $(\sum x_i)^2 = 63{,}680{,}400$;
$(\sum y_i)^2 = 31.92$; $r^2{}_{xy} = 0.371$.

Then, in the case of the combined through lanes, the K factor is only 37.1% linearly dependent upon the volume. The only conclusion to be drawn from these figures would be that the K factor assumes a slightly more linear dependence upon the volume when we consider the traffic stream as a whole rather than when we consider each individual lane.

Although the values of 26.1% and 37.1% are quite low, it was felt that a "best" straight line should be determined in any case. This line is calculated by the method of least squares and has the form:

$$y - \bar{y} = b_{y.x}\,(x-\bar{x})$$

where $\bar{x}$ and $\bar{y}$ are the means of the x's and the y's, and,

$$b_{y.x} = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum (x_i)^2 - (\sum x_i)^2} \qquad (15)$$

In the case of individual lanes,

$$\bar{x} = \frac{10{,}450}{12} = 870.83; \qquad \bar{y} = \frac{20.23}{12} = 1.69$$

and for the combined through lanes,

$$\bar{x} = \frac{7980}{4} = 1995; \qquad \bar{y} = \frac{5.65}{4} = 1.41$$

Substituting these values of $\bar{x}$ and $\bar{y}$ and the above values for $n, \sum x_i, \sum y_i, \sum x_i y_i, \sum (x_i)^2, (\sum x_i)^2$ in eq.(15), we respectively obtained:

$$b_{y.x} = 0.00079; \qquad \text{and } b_{y.x} = 0.00012.$$

Thus, the "best" straight lines for each individual lane and the combined through lanes are respectively:

$$y - 1.69 = 0.00079\,(x - 870.83) \qquad \text{(line A)}$$

$$y - 1.41 = 0.00012\,(x - 1995) \qquad \text{(line B)}$$

These lines have been plotted in Figures 14 and 15.

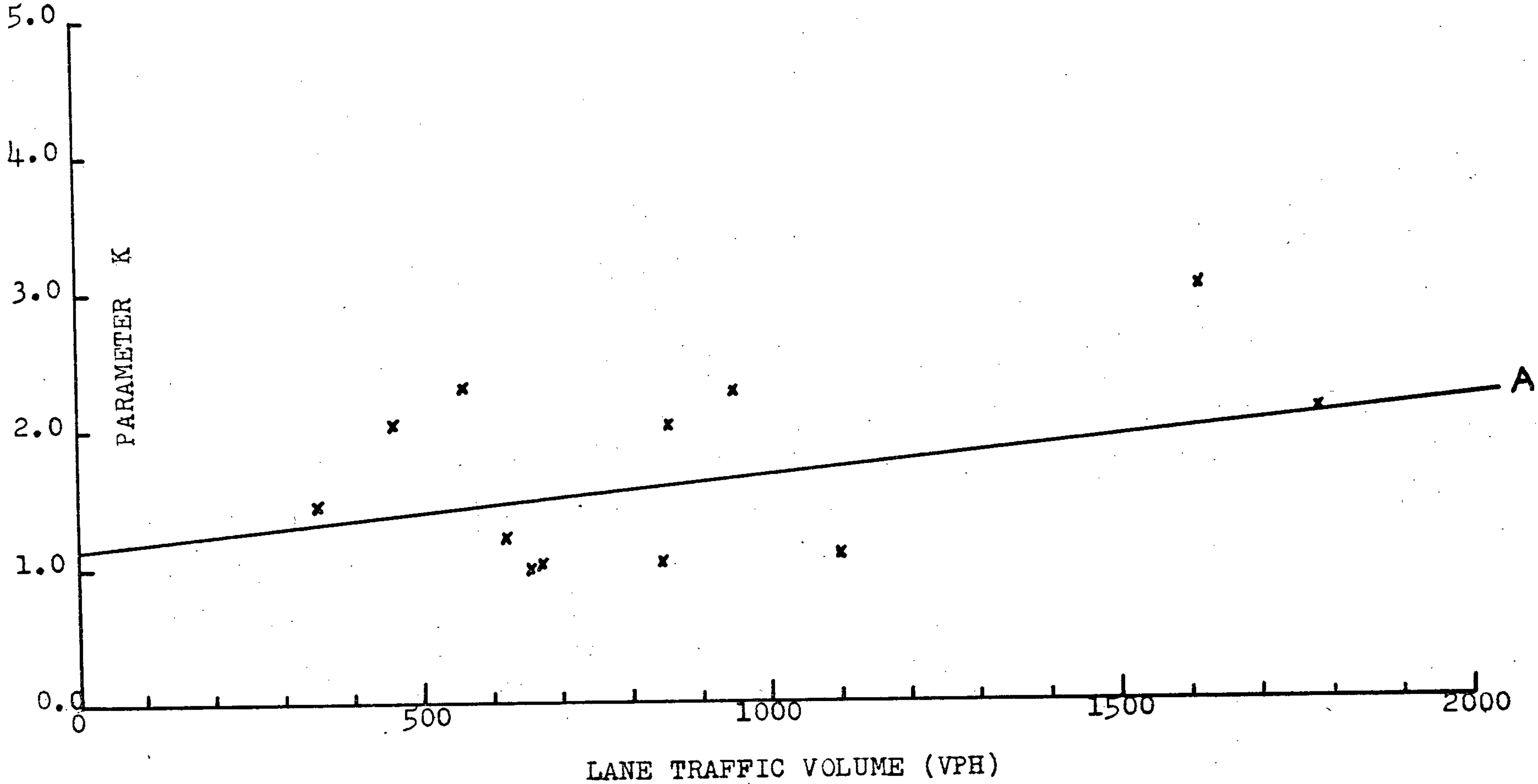


FIGURE 14: INFLUENCE OF LANE VOLUME ON PARAMETER K

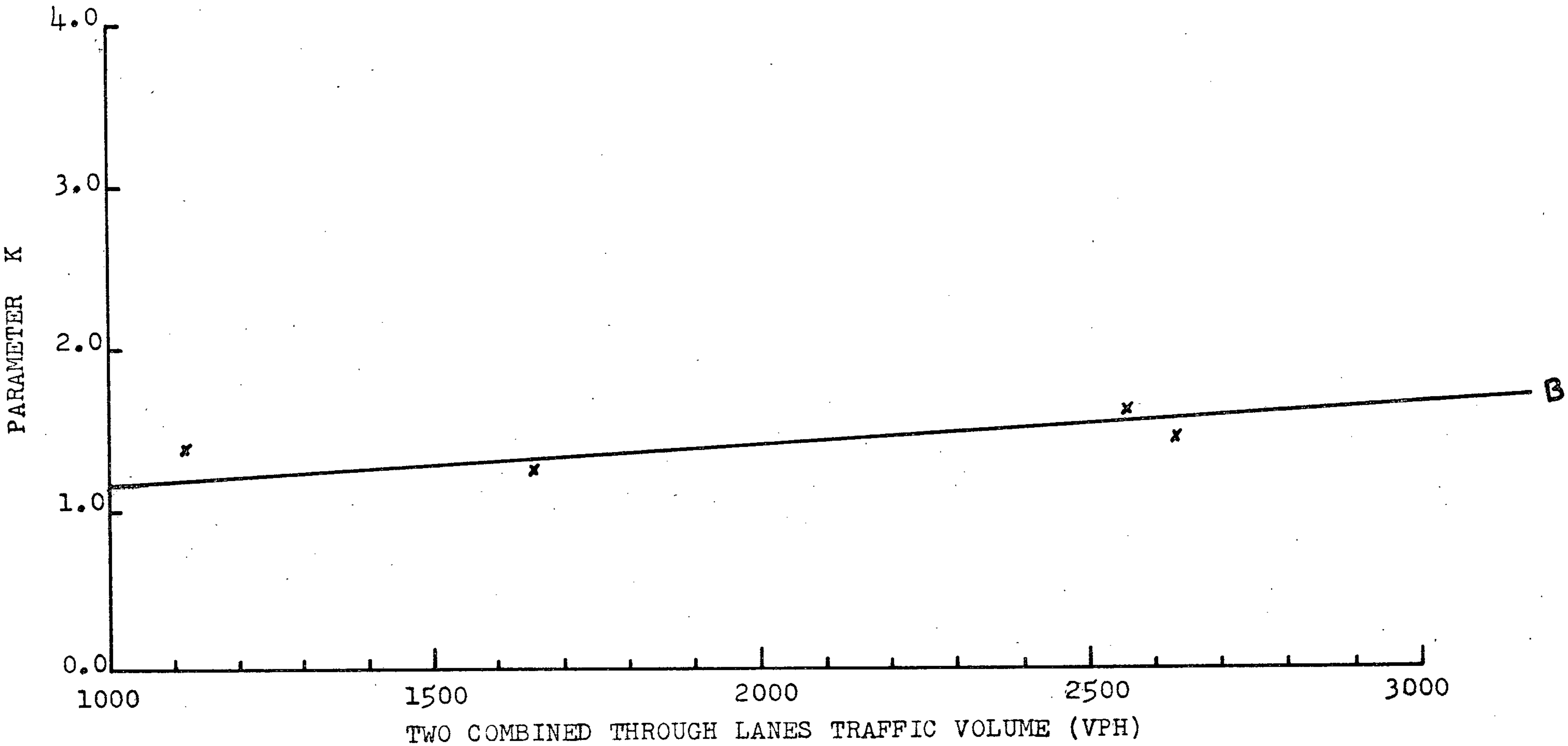


FIGURE 15: INFLUENCE OF TOTAL VOLUME OF COMBINED THROUGH LANES ON PARAMETER K

It should be emphasized that these two straight lines are not necessarily the best curves to fit the data but are merely the best "fitting" straight lines. We know from the values of the correlation coefficients that there is a very small linear relationship between the parameter K and the traffic volume. The true relationship is certainly a more elaborate one and would be represented by an equation of a higher degree. It was not felt however to be worthwhile to find out this true mathematical relationship since this would lead to a theory too elaborate for practical application. For all practical purposes, especially if accuracy is not required, we believe that these lines are reliable enough to show which value of K corresponding to a given volume should be used when working with a Gamma function.

RELATION OF PARAMETER K TO TYPE OF LANE

From what we have seen, we know that the relationship between the K values and the volumes of traffic is not a simple one. Let's now reclassify these K values, but this time in relation to the type of lane and the study location. Table 4 shows such a classification.

TABLE 4

CLASSIFICATION OF K VALUES IN RELATION TO TYPES OF LANE AND STUDY LOCATIONS

TYPE OF LANE	STUDY LOCATION				TOTAL
	Dufferin	Dixon Rd.	Avenue Rd.	Islington	
Deceleration	1.18	1.09	1.18	1.00	4.45
Driving	2.24	2.04	1.99	2.30	8.57
Passing	3.03	0.93	2.15	1.10	7.21
TOTAL	6.45	4.06	5.32	4.40	20.23

From observation of the above table, there appears to exist a relationship between the parameter K and the type of lane. We notice thatall values of K are close to unity for the deceleration lanes, and close to 2 for the driving lane; we observe no regularity in the case of the passing lanes. However, it is not certain that the type of lane is the only influencing factor. It is also possible that these values could have been influenced by the study location itself. The best way to determine the interaction of these factors was to carry out a well known statistical procedure, the analysis of variance.

Since the method of performing an analysis of variance can be found in every textbook of Statistics, it was not felt necessary to explain it here. A summary of this table is included, however, as well as the computations leading to our results. Table 5 shows the table used to analyse our data and the results are summarized in Table 6.

TABLE 5

GENERAL ANALYSIS OF VARIANCE TABLE.
TWO-WAY CLASSIFICATION WITH INTERACTION.

Source of Estimate	Sum of Squares S. of S.	Degrees of Freedom (D.F.)	Mean Square M.S.
MEAN	$\frac{T_{..}^2}{N}$	1	
BETWEEN ROWS	$\sum \frac{T_{.j}^2}{n_j} - \frac{T_{..}^2}{N}$	$n_i - 1$	S.of S./n_i-1
BETWEEN COLUMNS	$\sum \frac{T_{i.}^2}{n_i} - \frac{T_{..}^2}{N}$	$n_j - 1$	S.of S./n_j-1
INTERACTION	$\sum X_{ij}^2 - \sum \frac{T_{.j}^2}{n_j} - \sum \frac{T_{i.}^2}{n_i} + \frac{T_{..}^2}{N}$	$(n_i-1)(n_j-1)$	S.of S./$(n_i-1)(n_j-1)$
TOTAL	$\sum_{ij} X_{ij}^2$	$N = n_i n_j$	

where X_{ij} = the observations classified in n_i rows and n_j columns; $N = n_i n_j$; $T_{.j} = \sum_{n_j} X_{ij}$; $T_{i.} = \sum_{n_i} X_{ij}$; $T_{..} = \sum_{ij} X_{ij}$;

In our case (see Table 52), $n_i = 3$; $n_j = 4$; $N = 12$;

$\sum_{ij} X^2_{ij} = (1.18)^2 + \ldots\ldots + (1.10)^2 = 39.27$;

$\frac{T_{..}^2}{N} = \frac{(20.23)^2}{12} = 34.10$; $\sum \frac{T_{.j}^2}{n_j} = \frac{(4.45)^2}{4} + \frac{(8.57)^2}{4} + \frac{(7.21)^2}{4}$

$= 36.31$; $\sum \frac{T_{i.}^2}{n_i} = \frac{(6.45)^2}{3} + \frac{(4.06)^2}{3} + \frac{(5.32)^2}{3} + \frac{(4.40)^2}{3}$

$= 35.24$.

Thus the variation due to the type of lane = 36.31 - 34.10 = 2.21; the variation due to the study location = 35.24 - 34.10 = 1.14; the interaction = 39.27 - (34.10 + 2.21 + 1.14) = 1.82.

We can now set up the following analysis of variance table:

TABLE 6

ANALYSIS OF VARIANCE TABLE.
ALL THREE TYPES OF LANE INCLUDED.

SOURCE OF ESTIMATE	SUM OF SQUARE S. of S.	D.F.	MEAN SQUARE M.S.
Mean	34.10	1	
Between Types of Lane	2.21	2	2.21 ÷ 2 = 1.05
Between Study Locations	1.14	3	1.14 ÷ 3 = 0.38
Interaction	1.82	6	1.82 ÷ 6 = 0.303
Total	39.27	12	

The above results allow us to carry out a few tests of significance:

1. Let's check the hypothesis that the study location does not affect the factor K at the 5% level.

From our table, $\frac{0.38}{0.303} = 1.26$

From the F-distribution, $F(3,6,\ 0.05) = 4.76$

Since $1.26 < 4.76$, this means that the test is not significant and therefore the stated hypothesis is true. Thus the study location does not affect the K factor at the 5% level. This conclusion is very satisfactorily accepted since if the opposite had been true, it would have indicated that our results are simply due to chance and not conclusive in any way.

2. Let's now check the hypothesis that the type of lane does not affect the factor K at the 5% level.

From our table, $\frac{1.105}{0.303} = 3.64$

From the F-distribution, $F(2,6,0.05) = 5.14$

Since $3.64 < 5.14$, this means that the test is not significant and therefore the type of lane does not affect the K factor. In other words, the obtained results give no evidence of a relationship between the type of lane and the value of K. It need not be emphasized that this is a very disappointing conclusion since such a relationship had appeared strongly possible from observation of the results. We believe that the fact that the test did not turn significant can be explained by either one of the following causes: the number of observations was insufficient, thus providing no solid ground for a statistical analysis; or, what is more probable, the discrepancies between the K values of the passing lanes are so great that they may cause the test not to be significant. Although the first cause was quite possible and certainly cannot be eliminated, we felt that it was beyond the scope of this thesis to investigate it. The second cause could be simply checked by analysing the variance of the data for the driving and deceleration lanes, thus excluding the passing lanes where the greatest variations in the value of K were found. An analysis of variance was then carried out for the data of Table 7:

TABLE 7

CLASSIFICATION OF K VALUES IN RELATION TO STUDY LOCATIONS AND TWO TYPES OF LANE ONLY.

TYPE OF LANE	STUDY LOCATION				TOTAL
	Dufferin	Dixon Rd.	Avenue Rd.	Islington	
Deceleration	1.18	1.09	1.18	1.00	4.45
Driving	2.24	2.04	1.99	2.30	8.57
Total	3.42	3.13	3.17	3.30	13.02

In this case, $n_i = 2$; $n_j = 4$; $N = 8$;

$\sum_{ij} {}^2X_{ij} = (1.18)^2 + \ldots + (2.30)^2 = 23.40$; $\frac{T_{..}^2}{N} = \frac{(13.02)^2}{8}$

$= 21.19$; $\sum \frac{T_{.j}^2}{n_j} = \frac{(4.45)^2}{4} + \frac{(8.57)^2}{4} = 23.31$;

$\sum \frac{T_{i.}^2}{n_i} = \frac{(3.42)^2}{2} + \frac{(3.13)^2}{2} + \frac{(3.17)^2}{2} + \frac{(3.30)^2}{2} = 21.22$;

The variation due to type of lane = 23.31 - 21.19 = 2.12; the variation due to study location = 21.22 - 21.19 = 0.03; From this, the following analysis of variance table (Table

8) was set up:

TABLE 8

ANALYSIS OF VARIANCE TABLE.
PASSING LANE EXCLUDED.

SOURCE OF ESTIMATE	SUM OF SQUARE S. of S.	D.F.	MEAN SQUARE M.S.
Mean	21.19	1	
Between Types of Lane	2.12	1	$2.12 \div 1 = 2.12$
Between Study Locations	0.03	3	$0.03 \div 3 = 0.01$
Interaction	0.06	3	$0.06 \div 3 = 0.02$
Total	23.40	8	

We can now verify the same hypotheses as previously:

1. Hypothesis: the study location does not affect the parameter K at the 5% level.

From our table, $\frac{0.01}{0.02} = 0.5$

From the F-distribution, $F_{(3,3,0.05)} = 9.28$

Since $0.5 \ll 9.28$, the test is not significant. Therefore there is definitely no evidence to indicate that the study location affects the value of K.

2. Hypothesis: the type of lane does not affect the parameter K at the 5% level.

From our table, $\frac{2.12}{0.02} = 106$

From the F-distribution, $F_{(1,3,0.05)} = 10.13$

Since $106 \gg 10.13$, the type of lane is revealed as a highly significant effect. This is a very interesting conclusion that we had expected when we first observed Table 4 and suspected a trend between the type of lane and the factor K.

According to this, the headway distribution of an urban freeway would be theoretically represented by different distributions when we consider each lane separately. The deceleration lane headways would be best represented by an Erlang distribution with K=1, which is the Exponential or Poisson distribution; in the driving lane, a good theoretical model would be an Erlang distribution with K=2; finally, the passing lane assumes so many variations that a specific Erlang curve does not apply in all cases but any one of the family could apply in individual cases. These conclusions coincide well enough with what had already been obtained when performing the chi-square tests to compare the goodness of fit of Erlang (K=2) and Poisson distributions to our experimental data. These tests had first indicated a better fit with Erlang (K=2) than with Poisson in all driving lanes. In one occasion (Dixon Road), there was a statistical fit and one was very close (Islington). However, there is partial disagreement when we come to the deceleration lanes. Indeed, only one out of four deceleration lanes (it was at Avenue Rd.) had given a closer fit with Poisson when checking with the chi-square test. Two had showed a closer fit and one had given a statistical fit with Erlang (K=2), although in this last case a fit with Poisson was very close, the difference between the chi-square statistics and the tabulated value being as small as 2.90. These discrepancies are not easily explained but we think that the conclusions drawn from the analysis of variance table are much more reliable than those suggested by the chi-square tests. While the analysis of variance gives a positive evidence when it is significant, a significant chi-square test only provided us with a negative and indirect evidence if we consider the criteria that we used to decide which one of Erlang (K=2) or Poisson distribution best represented our experimental data. This criteria was based upon the minimum difference between the statistic obtained when fitting our data to either one of Poisson or Erlang and some tabulated values. This will be best illustrated by the following example: In the Islington deceleration lane, the inference that Erlang (K=2) gives a better fit than Poisson was based upon the fact that the difference between the statistic derived from Erlang (57.41) and the chi-square tabulated value (24.73) is smaller than the corresponding values (71.61 and 24.73 respectively) obtained when fitting our data to Poisson. We believe that such an inference still provides evidence but rather negative as compared with the evidence provided by an analysis of variance table. Since the closer fit to Erlang has also been decided according to the same criteria in the case of the Dixon Road deceleration lane, there is really only one case where the conclusions drawn from the analysis of variance table and the chi-square method basically disagree. This happens in the Dufferin deceleration lane for which a chi-square test provided significant for Poisson and not significant for Erlang (K=2). Here the previous attempts to explain the apparent contradiction between the conclusions inferred by both statistical methods do not hold since, being not significant, the chi-square test provides us with a positive evidence that we cannot attenuate. Although the difference between the statistic obtained when fitting our data to Poisson and the tabulated value is very small (2.90), the test with Poisson is not significant and therefore, even if this gives ground to infer that Poisson nearly gives a fit, the conclusion suggested by the non-significant test is much more positive and must be the one accepted.

Thus, with the exception of this last case, the apparent discrepancies between the trends suggested by the analysis of variance and chi-square methods are not serious and can be partly explained. Therefore, we have good reasons to affirm that there is a relationship between the value of the parameter K and the individual lanes of an urban freeway, as demonstrated by the previous analysis of variance. It has also been suggested to apply Poisson in the case of a deceleration lane and Erlang (K=2) in the case of a driving lane. It should be emphasized however that they are not necessarily the distributions which will give the best fit in all individual cases. The best fitting distribution would be one with a value of K calculated in each case from the experimental data. For all practical purposes however, we believe that the distributions with K=1 and K=2 should give the best overall fit when applied to deceleration and driving lanes respectively.

IV. CONCLUSIONS

This study has attempted to investigate the distribution of headways on Highway 401, an urban freeway. The statistical approach was an attempt tomake the analysis general enough so that the results could have application to other locations. It is hoped that the observed headway distributions on Highway 401 and their statistical analysis can be extended and applied to the longitudinal flow pattern of any urban freeway. The inferences drawn from our study should certainly be supplemented by more data, but nevertheless they do establish a foundation for further investigation.

The analysis performed in this thesis suggested the following conclusions:

1. The photographic method, although it is accurate and perhaps the most useful tool in a comprehensive study of the traffic flow, is very tedious when extracting the data from the films. Another method might be used more advantageously when studying only the longitudinal traffic flow characteristics.

2. The headway distribution of an urban freeway shows many discrepancies with the Poisson and Erlang (K=2) probability distributions. However, the Erlang distribution

generally gives a closer statistical fit to observed data than Poisson and should therefore be preferred to the latter to describe the pattern of arrivals on a 4-lane urban freeway. For all practical purposes, calculations based on an Erlang (K=2) distribution are mostly in the safe side and certainly more accurate than when based on a Poisson distribution. Our present traffic theory actually based on Poisson would undoubtedly gain in accuracy if it was modified and based on Erlang (K=2).

3. The Erlang distribution fits the observed distribution of headways most closely when such a distribution is considered for the combined through lanes rather than for each lane taken individually.

4. The probability of a linear relationship between the traffic volume and the parameter K of the Erlang distribution is very small. This probability appears to increase when the traffic lanes are considered in combination instead of individually. The assumption of a linear relationship however is very useful and could be advantageously used when great accuracy is not required. A Gamma function should then be used, the value of the parameter K being taken from either one of the Fig. 14 or 15.

5. An interesting result is that there appears to exist a correlation between the type of lane and the parameter K. Accordingly, an Erlang distribution with K=2

would generally apply to the headway distribution in the driving lane and an Erlang distribution with K=1 (or Poisson) would apply to the deceleration lane. This was expected in the latter case since the vehicles diverging in the deceleration lane are likely drawn at random from the main through traffic stream, thus suggesting the use of the Poisson distribution. The fact that Poisson did not give a fit in this case could be explained by the fact that we don't have a "complete randomness" since the diverging vehicles were influenced by the through traffic stream before performing their diverging manoeuver.

6. The "two straight lines theory" as suggested by Greenshields (2) did not apply to Highway 401. It would therefore appear that his theory cannot be generalized to all conditions, or should certainly be modified.

7. A model of hyper-exponential distribution has been tried but proved unsuccessful to fit the observed data. It is felt however that further investigation with a different model could turn out more successful and should therefore be carried out.

8. A statistical model, by nature, isa mathematical formula which intends to describe some observed data with the greatest accuracy possible. It is therefore impossible that it fits perfectly the data, otherwise it becomes a physical law based on certainty and can no longer be called

TABLE A-9

OBSERVED AND EXPECTED DISTRIBUTIONS OF HEADWAYS.
DECELERATION LANE - DUFFERIN

CLASS INTERVAL (SEC)	FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	1	1	99.4	91.3	98.5
1.0-1.9	13	14	91.9	83.4	94.8
2.0-2.9	15	29	83.2	76.1	89.6
3.0-3.9	21	50	71.1	69.5	83.5
4.0-4.9	8	58	66.5	63.4	76.9
5.0-5.9	13	71	59.0	57.9	70.2
6.0-6.9	10	81	53.2	52.9	63.6
7.0-7.9	6	87	49.7	48.3	57.3
8.0-8.9	12	99	42.8	44.1	51.3
9.0-9.9	6	105	39.3	40.3	45.7
10.0-10.9	7	112	35.3	36.8	40.6
11.0-11.9	8	120	30.6	33.6	35.9
12.0-12.9	4	124	28.3	30.6	31.6
13.0-13.9	5	129	25.4	28.0	27.8
14.0-14.9	7	136	21.4	25.5	24.3
≥15.0	37	173	0.0	-	-
	N=173				

TABLE A-10

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
DRIVING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	3	3	98.5	79.0	91.2
1.0-1.9	49	52	74.0	62.4	75.6
2.0-2.9	43	95	52.5	49.3	58.6
3.0-3.9	28	123	38.5	38.9	43.7
4.0-4.9	21	144	28.0	30.7	31.7
5.0-5.9	11	155	22.5	24.3	22.6
6.0-6.9	15	170	15.0	19.2	15.8
7.0-7.9	12	182	9.0	15.1	10.9
8.0-8.9	5	187	6.5	12.0	7.5
9.0-9.9	4	191	4.5	9.4	5.1
10.0-10.9	1	192	4.0	7.5	3.5
11.0-11.9	2	194	3.0	5.9	2.3
12.0-12.9	0	194	3.0	4.6	1.6
13.0-13.9	2	196	2.0	3.7	1.0
14.0-14.9	2	198	1.0	2.9	0.7
15.0-15.9	1	199	0.5	2.3	0.4
16.0-16.9	1	200	0.0	1.8	0.3
	N=200				

TABLE A-11

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
PASSING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	14	14	93.0	60.8	73.7
1.0-1.9	116	130	35.0	36.9	40.8
2.0-2.9	43	173	13.5	22.5	20.1
3.0-3.9	14	187	6.5	13.6	9.3
4.0-4.9	6	193	3.5	8.3	4.1
5.0-5.9	2	195	2.5	5.0	1.7
6.0-6.9	2	197	1.5	3.1	0.7
7.0-7.9	2	199	0.5	1.9	0.3
8.0-8.9	0	199	0.5	1.1	0.1
9.0-9.9	0	199	0.5	0.7	0.05
10.0-10.9	0	199	0.5	0.4	0.02
11.0-11.9	0	199	0.5	0.3	0.009
12.0-12.9	1	200	0.0	0.2	0.003
	N=200				

TABLE A-12

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
DECELERATION LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	1	1	99.5	84.2	95.3
1.0-1.9	50	51	74.5	70.9	84.8
2.0-2.9	35	86	57.0	59.7	72.4
3.0-3.9	28	114	43.0	50.3	60.0
4.0-4.9	11	125	37.5	42.3	48.7
5.0-5.9	20	145	27.5	35.6	38.9
6.0-6.9	7	152	24.0	30.0	30.7
7.0-7.9	11	163	19.5	25.3	23.9
8.0-8.9	5	168	16.0	21.3	18.5
9.0-9.9	3	171	14.5	17.9	14.3
10.0-10.9	4	175	12.5	15.1	10.9
11.0-11.9	2	177	11.5	12.7	8.3
12.0-12.9	4	181	9.5	10.7	6.2
13.0-13.9	2	183	8.5	9.0	4.7
14.0-14.9	3	186	7.0	7.6	3.5
⩾ 15	14	200	0.0	-	-
	N=200				

TABLE A-13

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
COMBINED THROUGH LANES - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	47	47	76.5	73.3	87.1
1.0-1.9	59	106	47.0	53.7	64.7
2.0-2.9	29	135	32.5	39.3	44.3
3.0-3.9	15	150	25.0	28.8	29.0
4.0-4.9	13	163	18.5	21.1	18.3
5.0-5.9	10	173	13.5	15.5	11.4
6.0-6.9	5	178	11.0	11.3	6.9
7.0-7.9	5	183	8.5	8.3	4.1
8.0-8.9	9	192	4.0	6.1	2.4
9.0-9.9	3	195	2.5	4.5	1.4
10.0-10.9	2	197	1.5	3.3	0.9
11.0-11.9	2	199	0.5	2.4	0.5
12.0-12.9	0	199	0.5	1.8	0.3
13.0-13.9	1	200	0.0	1.3	0.2
	N=200				

TABLE A-14

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
COMBINED THROUGH LANES - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	44	44	78.0	63.1	76.4
1.0-1.9	75	119	40.5	39.8	45.0
2.0-2.9	33	152	24.0	25.1	23.7
3.0-3.9	18	170	15.0	15.8	11.7
4.0-4.9	9	179	10.5	10.0	5.6
5.0-5.9	9	188	6.0	6.3	2.9
6.0-6.9	6	194	3.0	4.0	1.2
7.0-7.9	1	195	2.5	2.5	0.5
8.0-8.9	2	197	1.5	1.6	0.2
9.0-9.9	2	199	0.5	1.0	0.1
10.0-10.9	0	199	0.5	0.6	0.04
11.0-11.9	1	200	0.0	0.4	0.02
	N=200				

TABLE A-15

OBSERVED AND EXPECTED DISTRIBUTIONS OF HEADWAYS.
COMBINED THROUGH LANES - DUFFERIN STREET

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	61	61	69.5	49.1	58.4
1.0-1.9	87	148	26.0	24.1	22.3
2.0-2.9	33	181	9.5	11.8	7.4
3.0-3.9	14	195	2.5	5.8	2.2
4.0-4.9	3	198	1.0	2.8	0.7
5.0-5.9	1	199	0.5	1.4	0.2
6.0-6.9	0	199	0.5	0.7	0.05
7.0-7.9	0	199	0.5	0.3	0.01
8.0-8.9	1	200	0.0	0.2	0.004
	N=200				

TABLE A-16

OBSERVED AND EXPECTED DISTRIBUTIONS
OF HEADWAYS.
COMBINED THROUGH LANES - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	CUMULATIVE FREQUENCY	CUMULATIVE FREQUENCY 100%	% OF HEADWAYS GREATER THAN THE CLASS INTERVAL UPPER LIMIT SHOWN	
				Theoretical Poisson	Theoretical Erlang (K=2)
0.0-0.9	62	62	69.0	48.1	56.9
1.0-1.9	95	157	21.5	23.1	21.0
2.0-2.9	26	183	8.5	11.1	6.6
3.0-3.9	9	192	4.0	5.3	1.9
4.0-4.9	4	196	2.0	2.6	0.6
5.0-5.9	2	198	1.0	1.2	0.2
6.0-6.9	1	199	0.5	0.6	0.05
7.0-7.9	1	200	0.0	0.3	0.01
	N=200				

APPENDIX "B"

Curves of observed and theoretical distributions of Headways at all study locations.

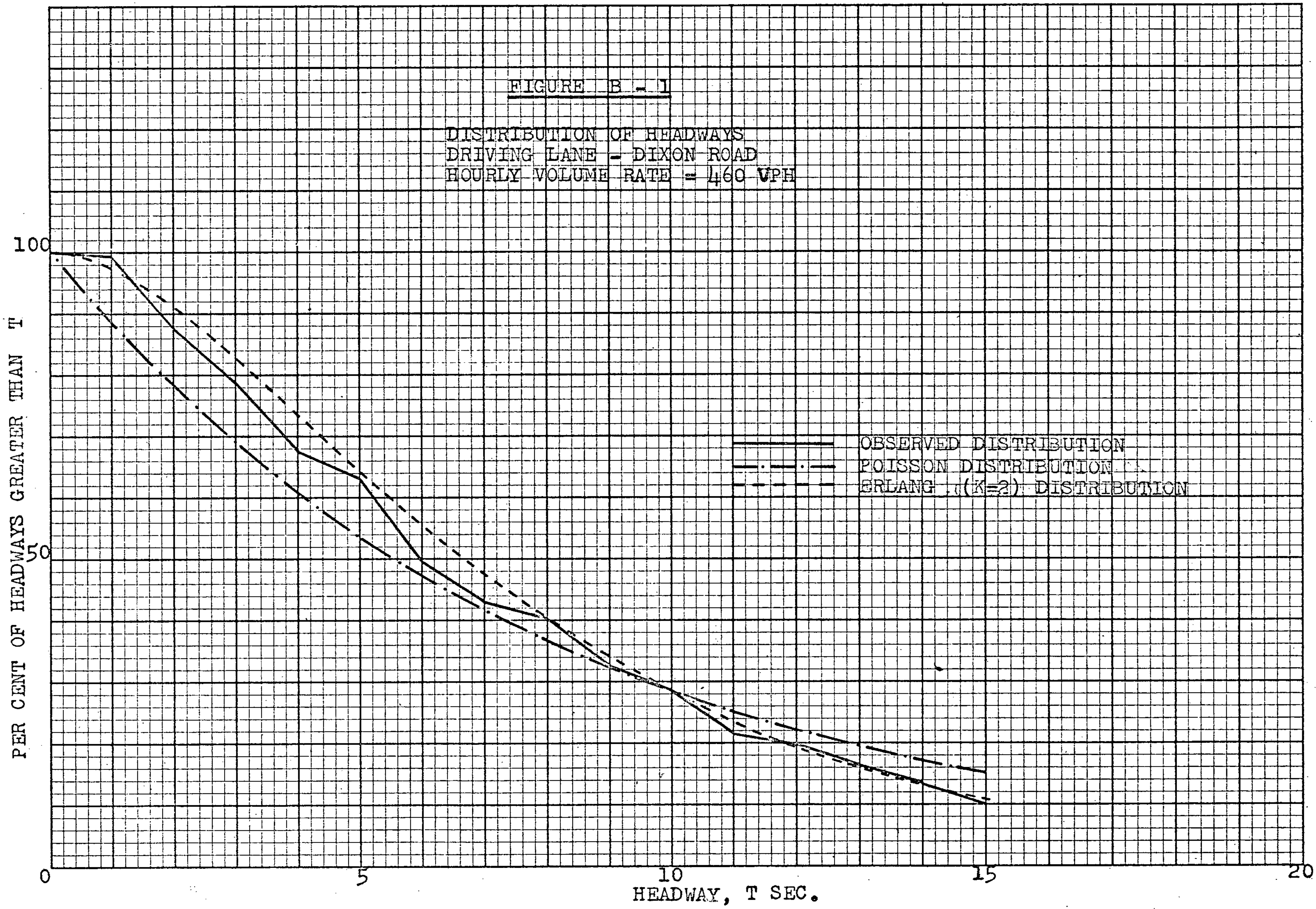


FIGURE B - 1
DISTRIBUTION OF HEADWAYS
DRIVING LANE - DIXON ROAD
HOURLY VOLUME RATE = 460 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
100
50
0
5
10
15
20
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.

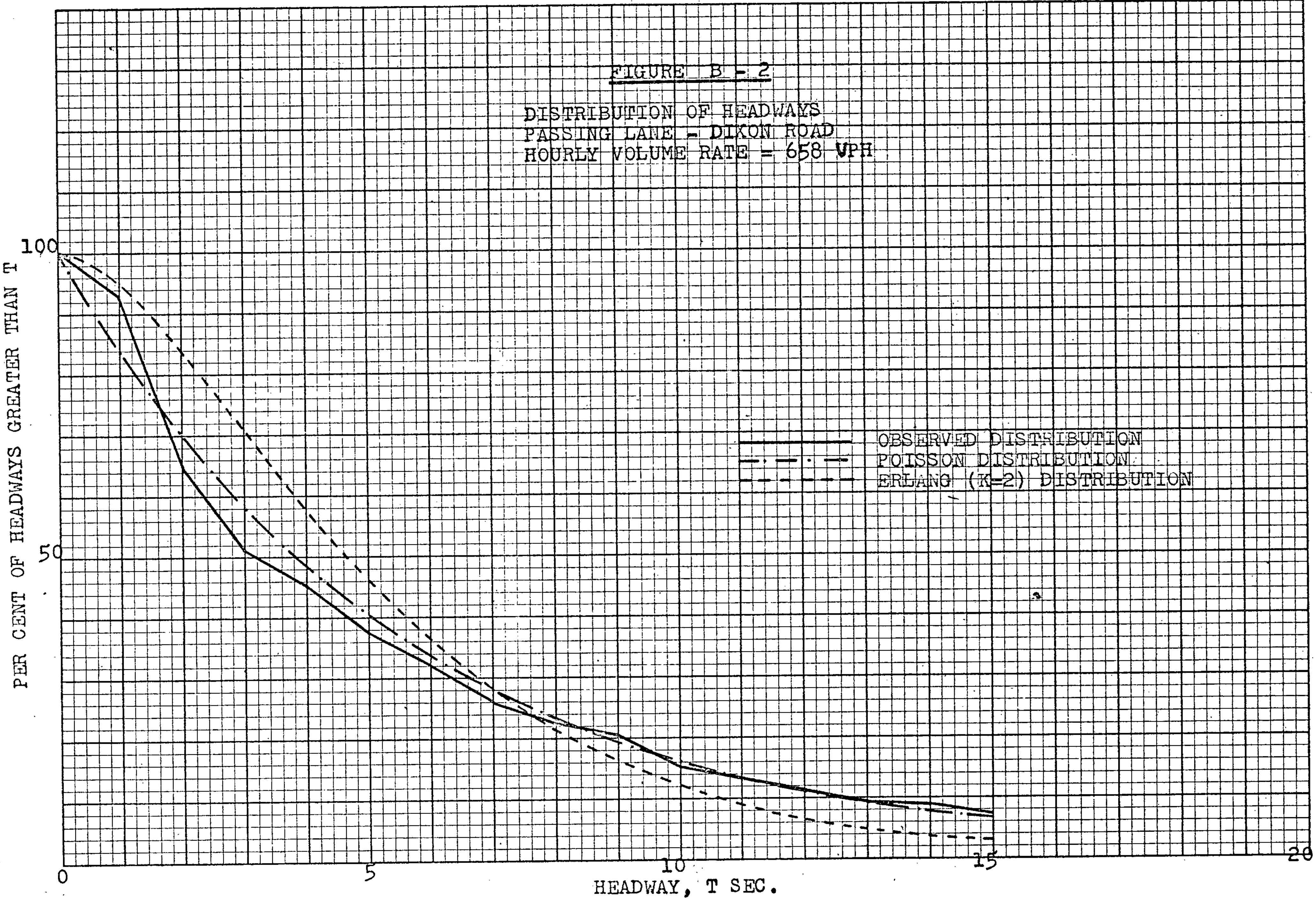
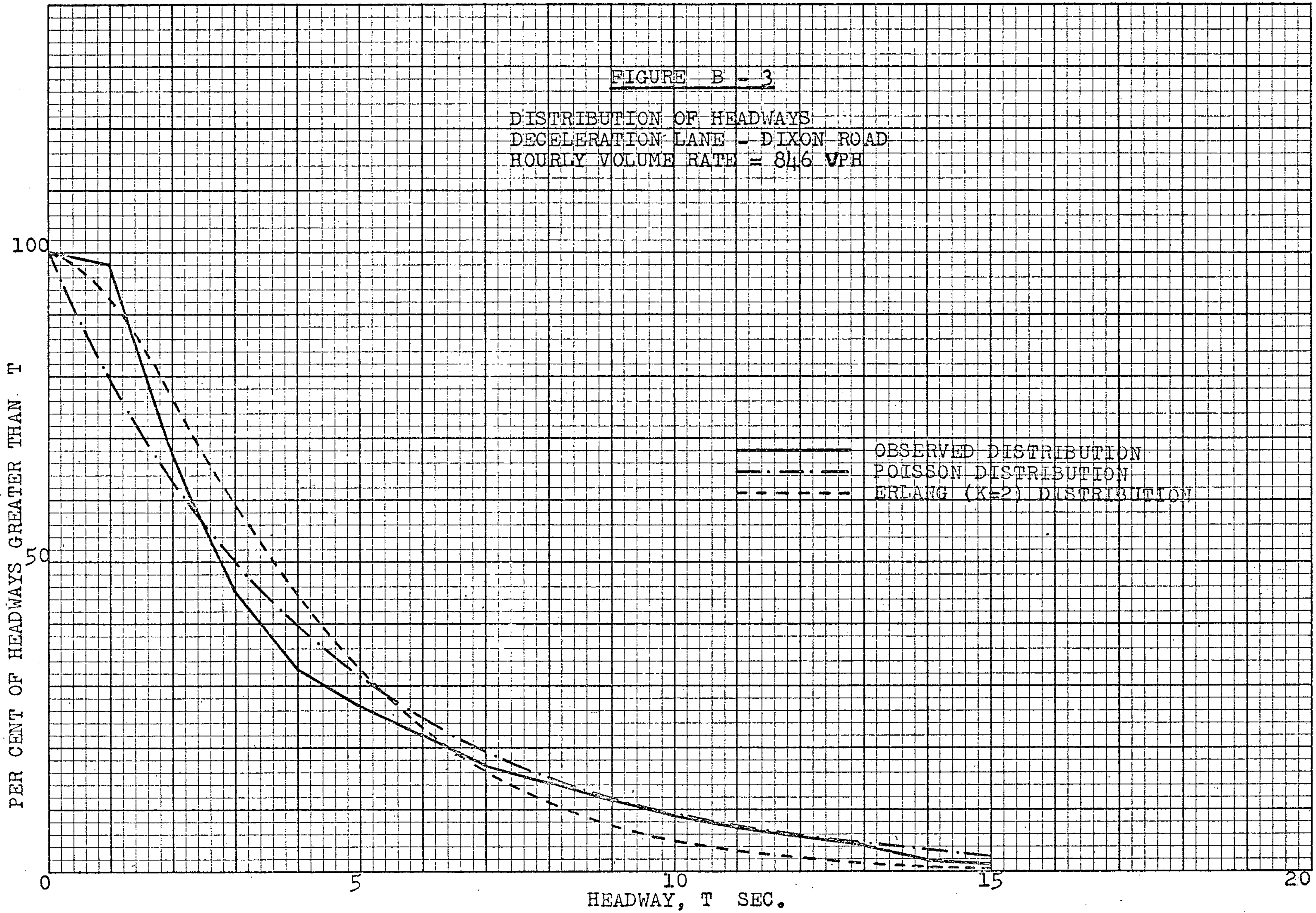


FIGURE B - 2
DISTRIBUTION OF HEADWAYS
PASSING LANE - DIXON ROAD
HOURLY VOLUME RATE = 658 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

MICROGRAPH G9-6 10 X 10 to the inch, 5th lines heavy MADE IN CANADA



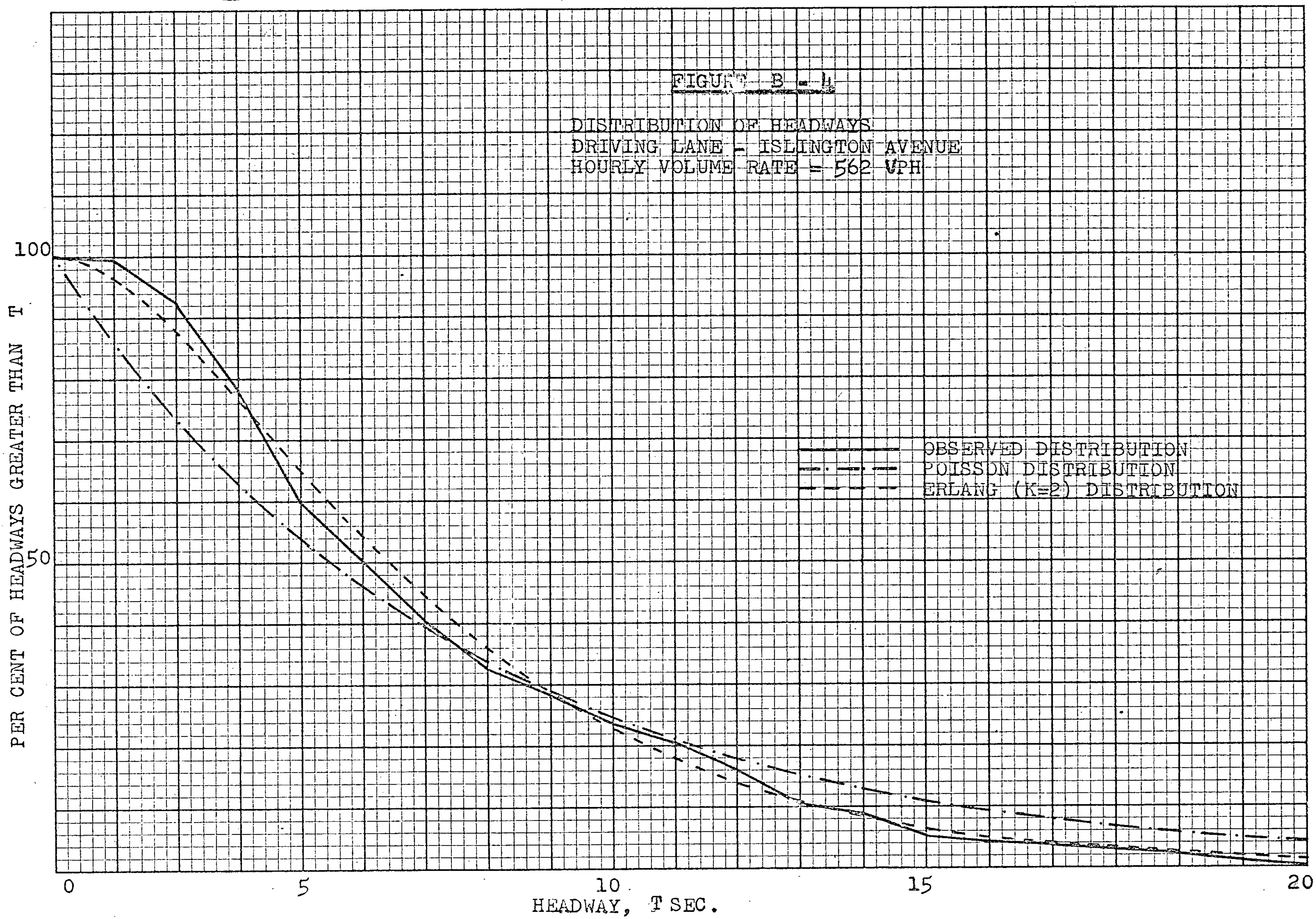


FIGURE B - 4
DISTRIBUTION OF HEADWAYS
DRIVING LANE - ISLINGTON AVENUE
HOURLY VOLUME RATE = 562 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
100
50
PER CENT OF HEADWAYS GREATER THAN T
0
5
10
15
20
HEADWAY, T SEC.

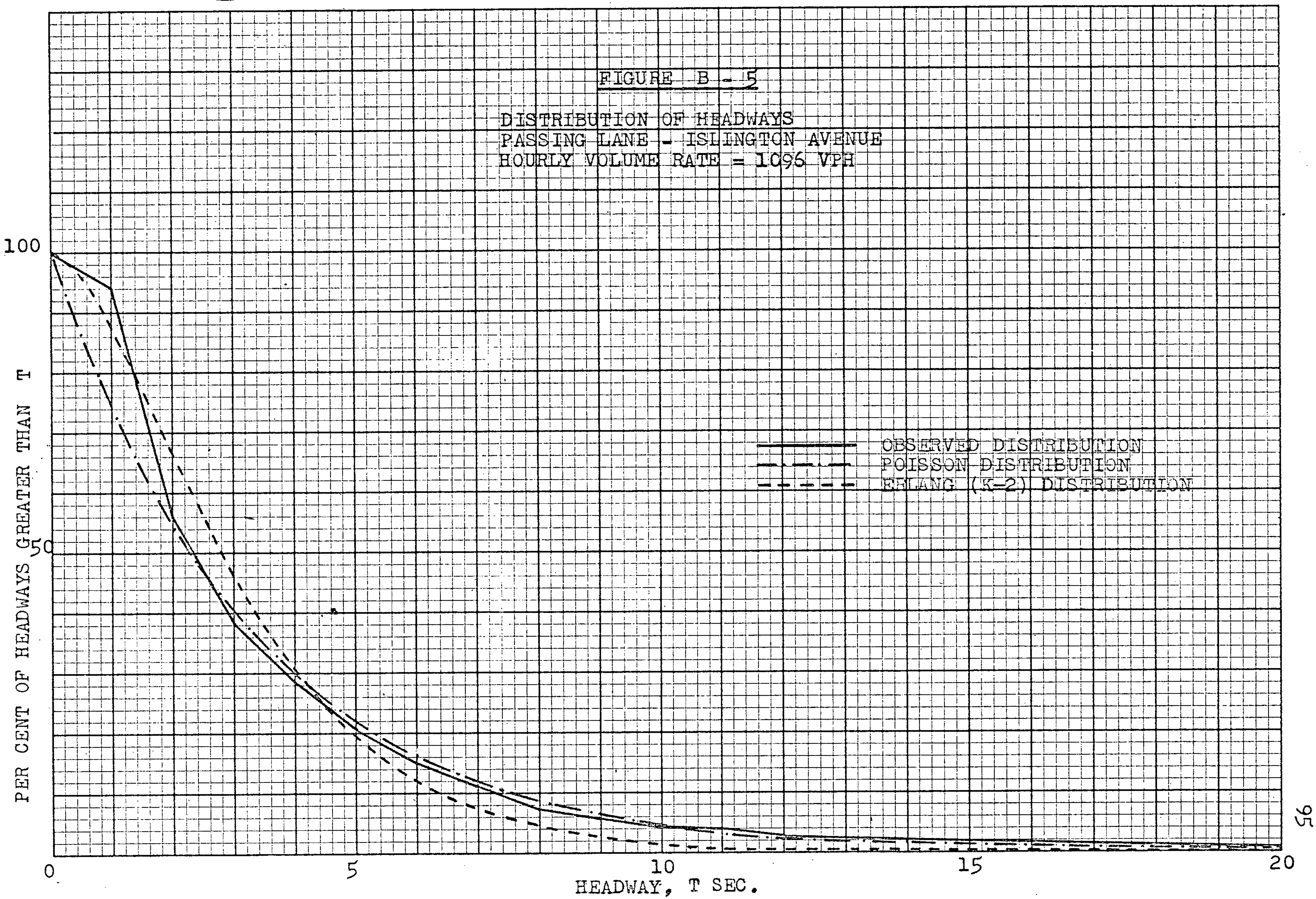


FIGURE B - 5
DISTRIBUTION OF HEADWAYS
PASSING LANE - ISLINGTON AVENUE
HOURLY VOLUME RATE = 1096 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
100
50
PER CENT OF HEADWAYS GREATER THAN T
0
5
10
15
20
HEADWAY, T SEC.

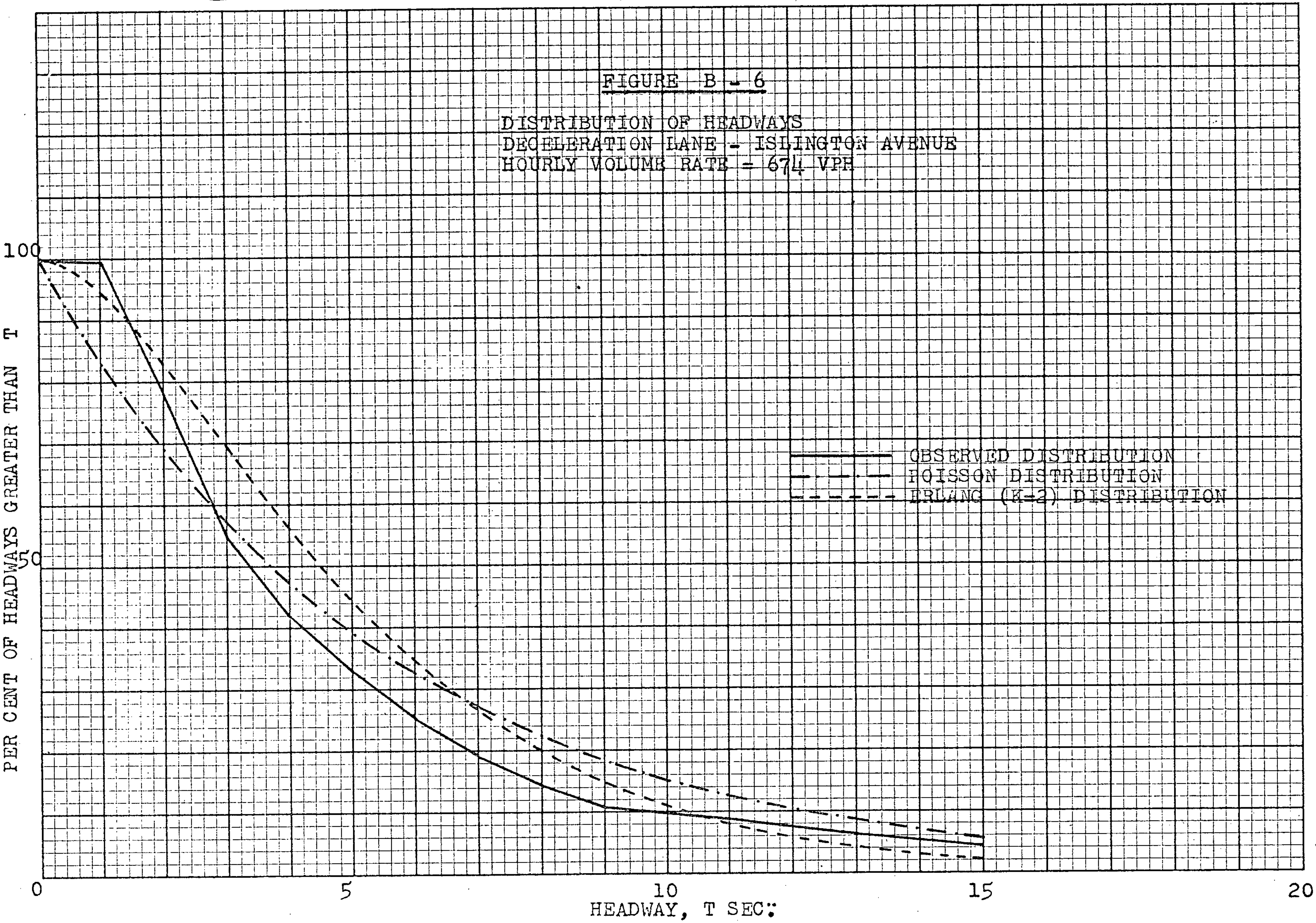


FIGURE B - 6
DISTRIBUTION OF HEADWAYS
DECELERATION LANE - ISLINGTON AVENUE
HOURLY VOLUME RATE = 674 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

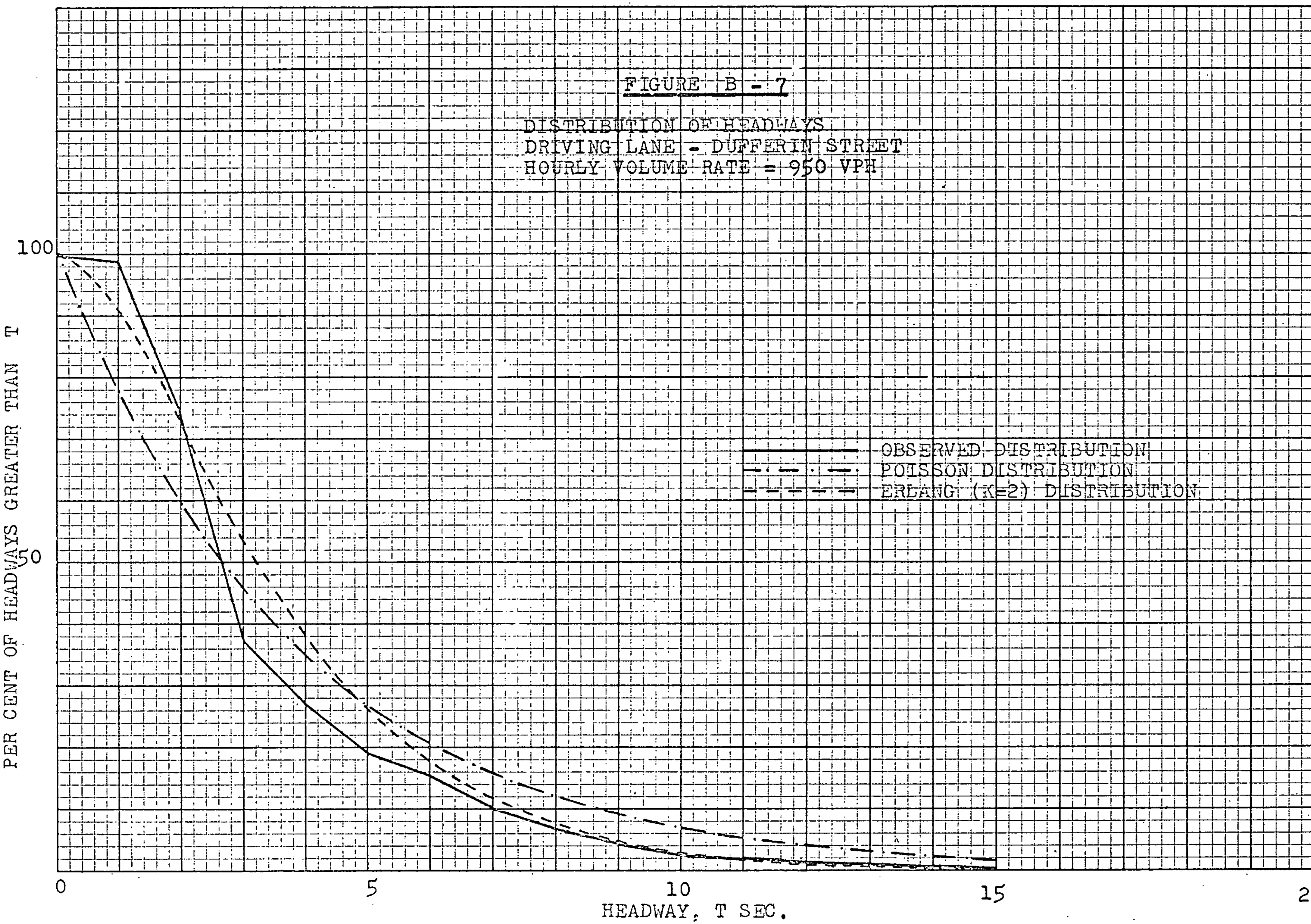
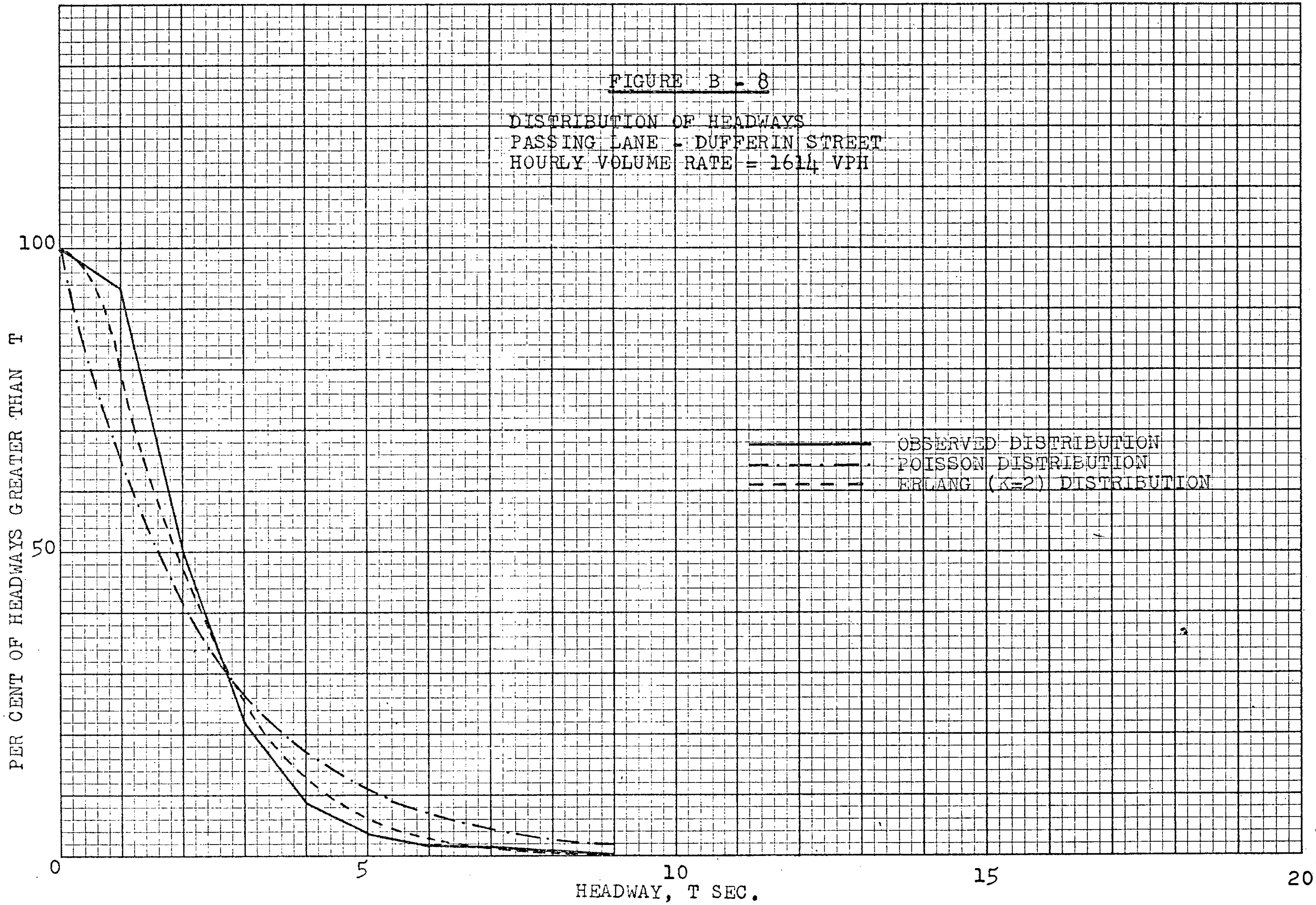


FIGURE B - 7
DISTRIBUTION OF HEADWAYS
DRIVING LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 950 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

FIGURE B - 8

DISTRIBUTION OF HEADWAYS
PASSING LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 1614 VPH



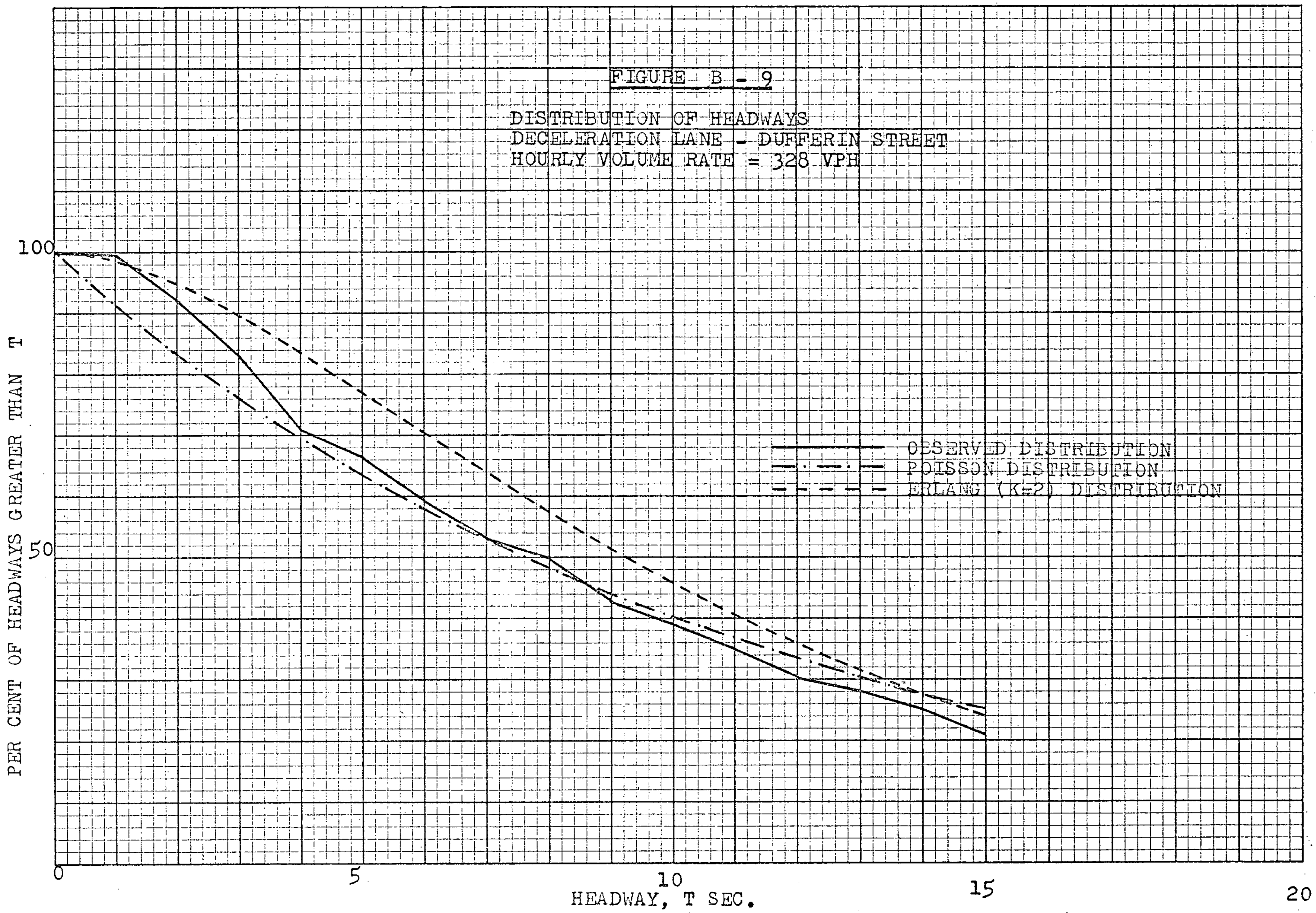


FIGURE B - 9
DISTRIBUTION OF HEADWAYS
DECELERATION LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 328 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

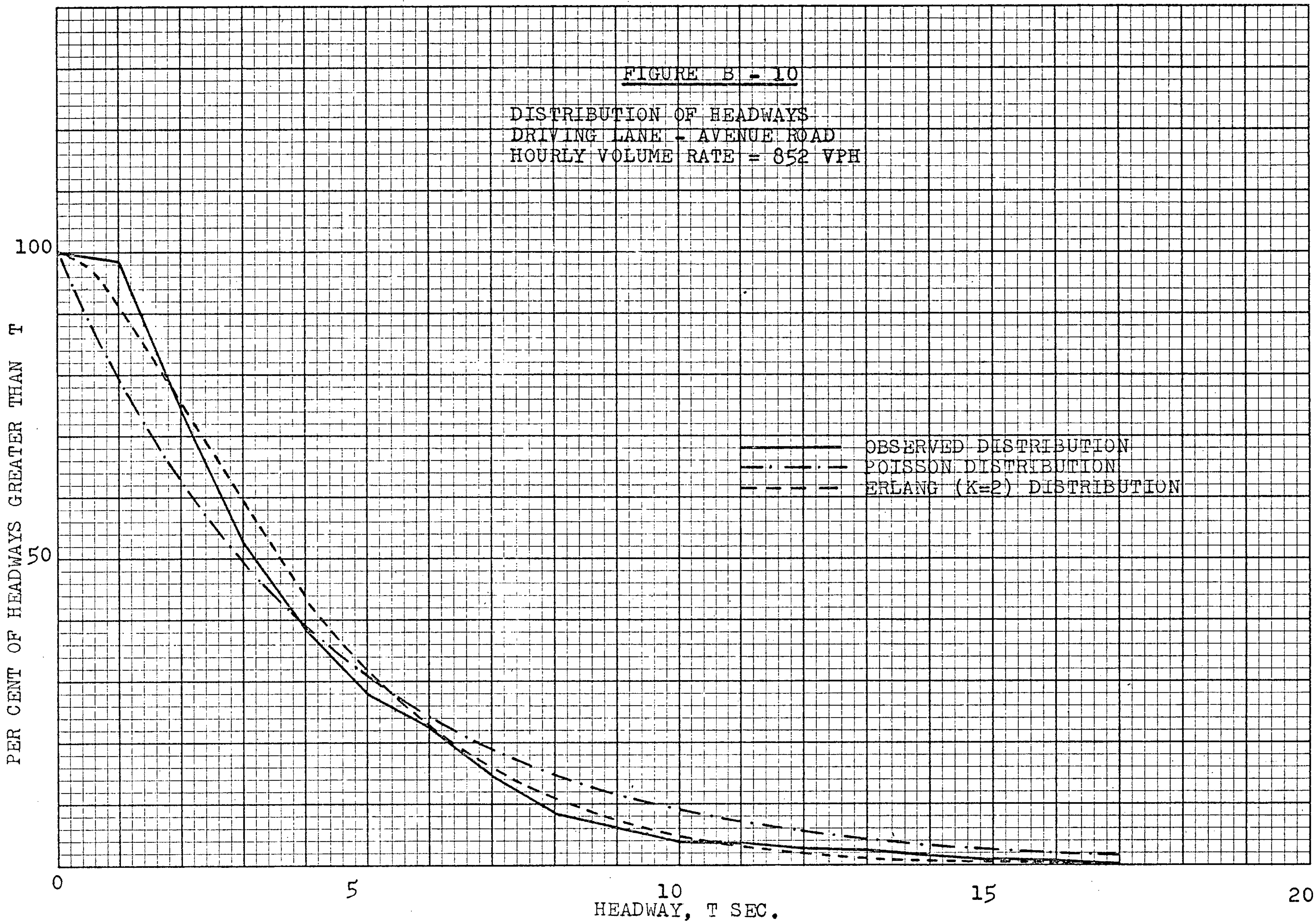
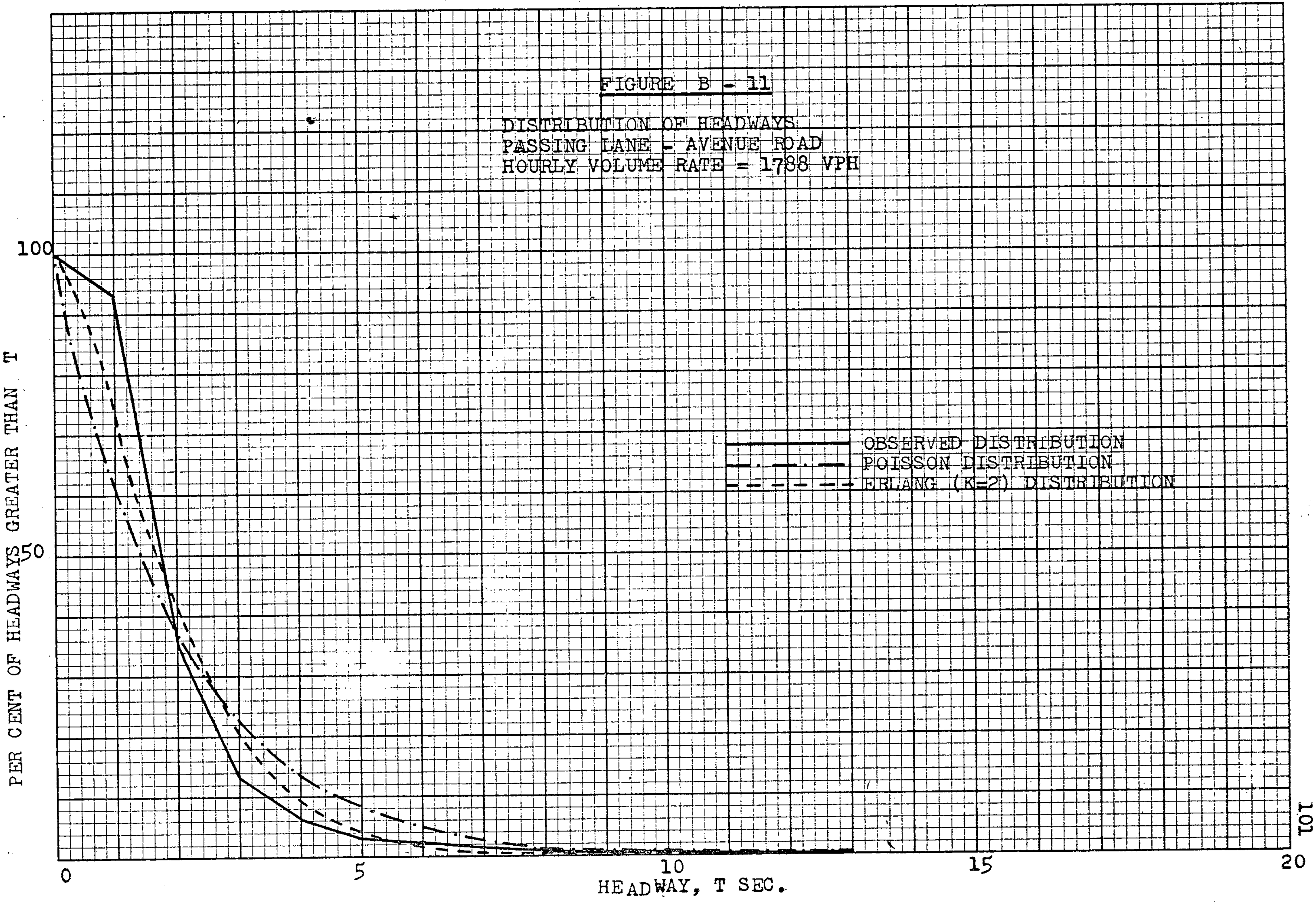


FIGURE B - 10
DISTRIBUTION OF HEADWAYS
DRIVING LANE - AVENUE ROAD
HOURLY VOLUME RATE = 852 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

FIGURE B - 11

DISTRIBUTION OF HEADWAYS
PASSING LANE - AVENUE ROAD
HOURLY VOLUME RATE = 1788 VPH



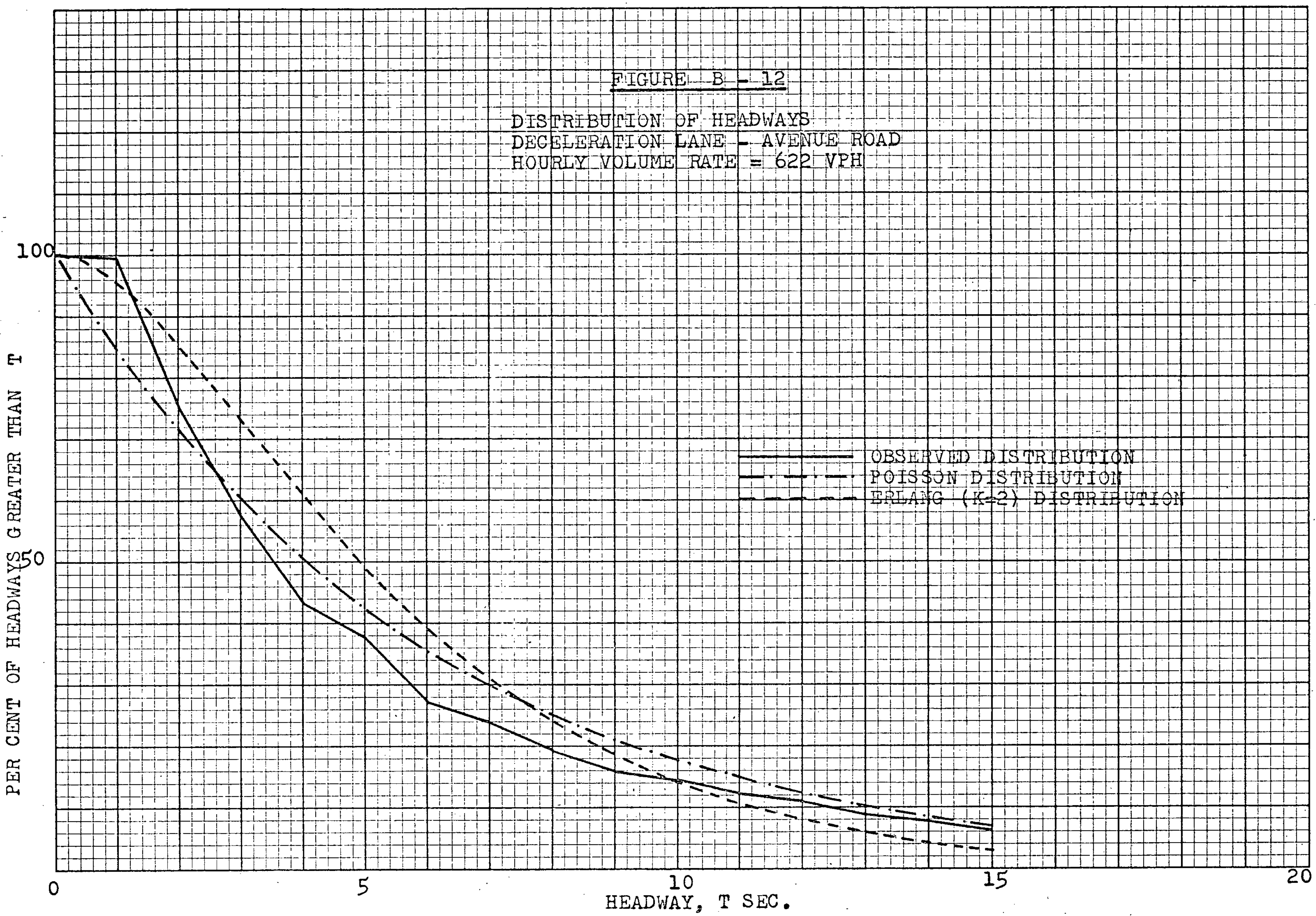


FIGURE B - 12
DISTRIBUTION OF HEADWAYS
DECELERATION LANE - AVENUE ROAD
HOURLY VOLUME RATE = 622 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
100
50
PER CENT OF HEADWAYS GREATER THAN T
0
5
10
15
20
HEADWAY, T SEC.

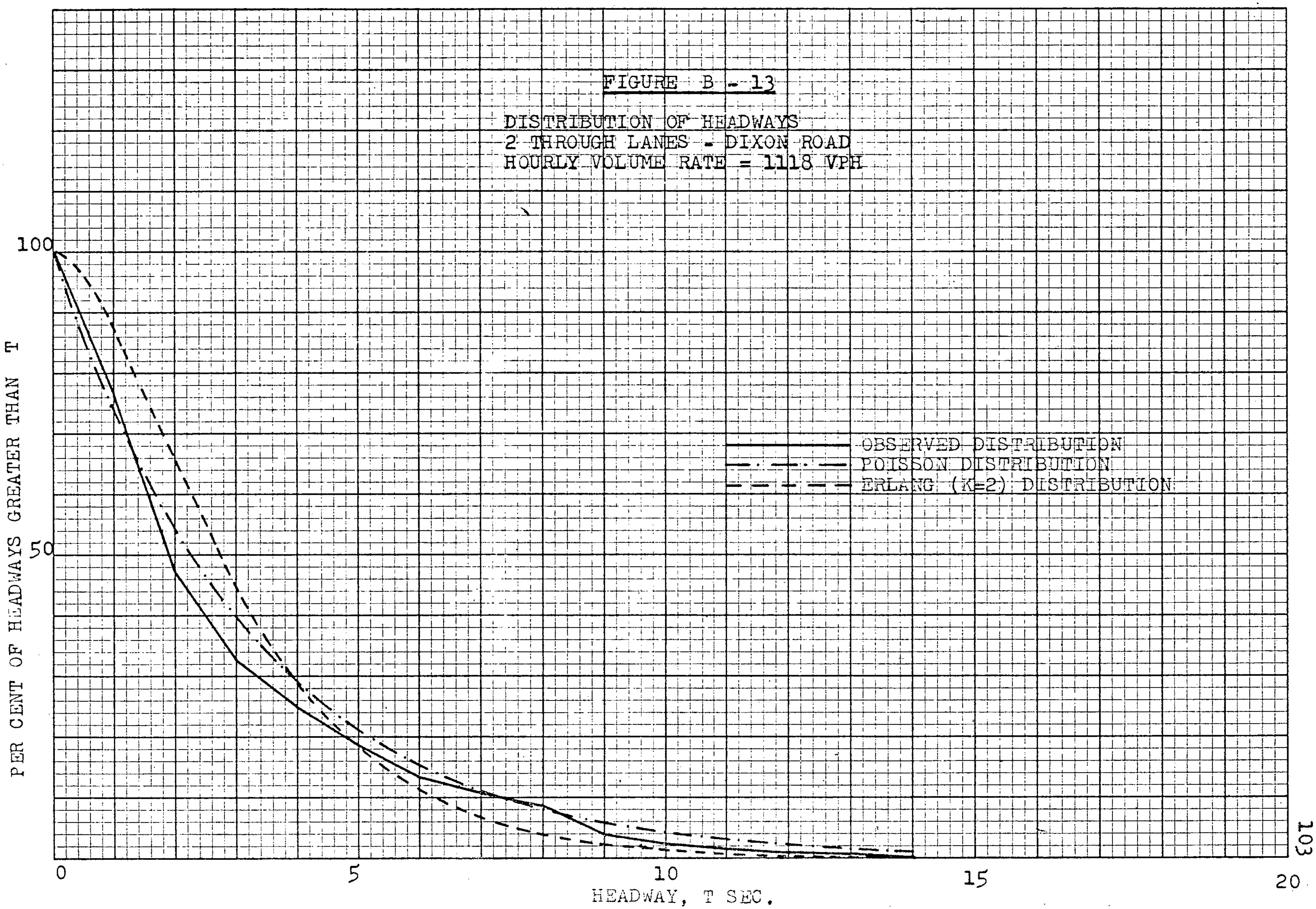


FIGURE B - 13
DISTRIBUTION OF HEADWAYS
2 THROUGH LANES - DIXON ROAD
HOURLY VOLUME RATE = 1118 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
100
50
PER CENT OF HEADWAYS GREATER THAN T
0
5
10
15
20
HEADWAY, T SEC.

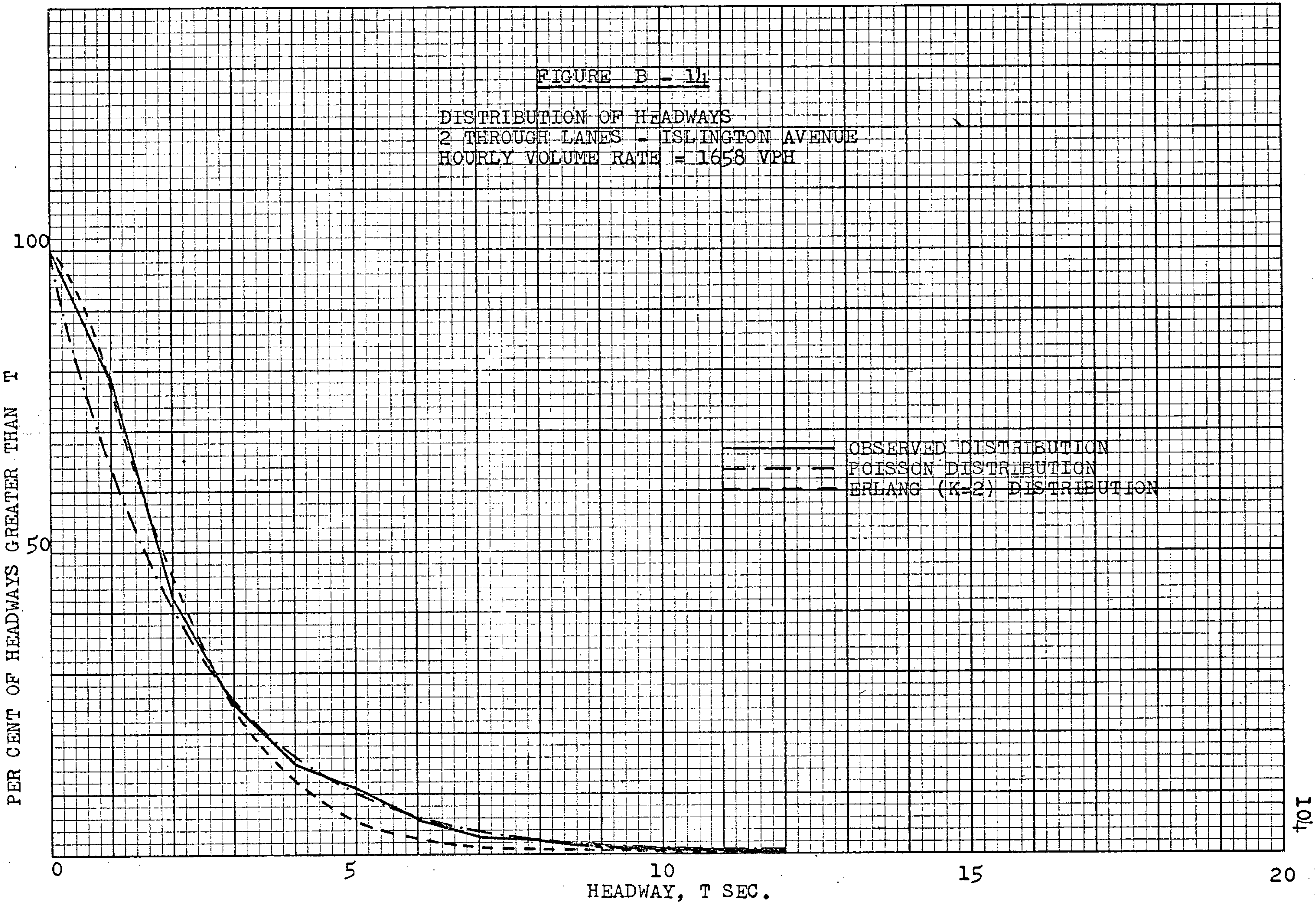


FIGURE B - 14
DISTRIBUTION OF HEADWAYS
2 THROUGH LANES - ISLINGTON AVENUE
HOURLY VOLUME RATE = 1658 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

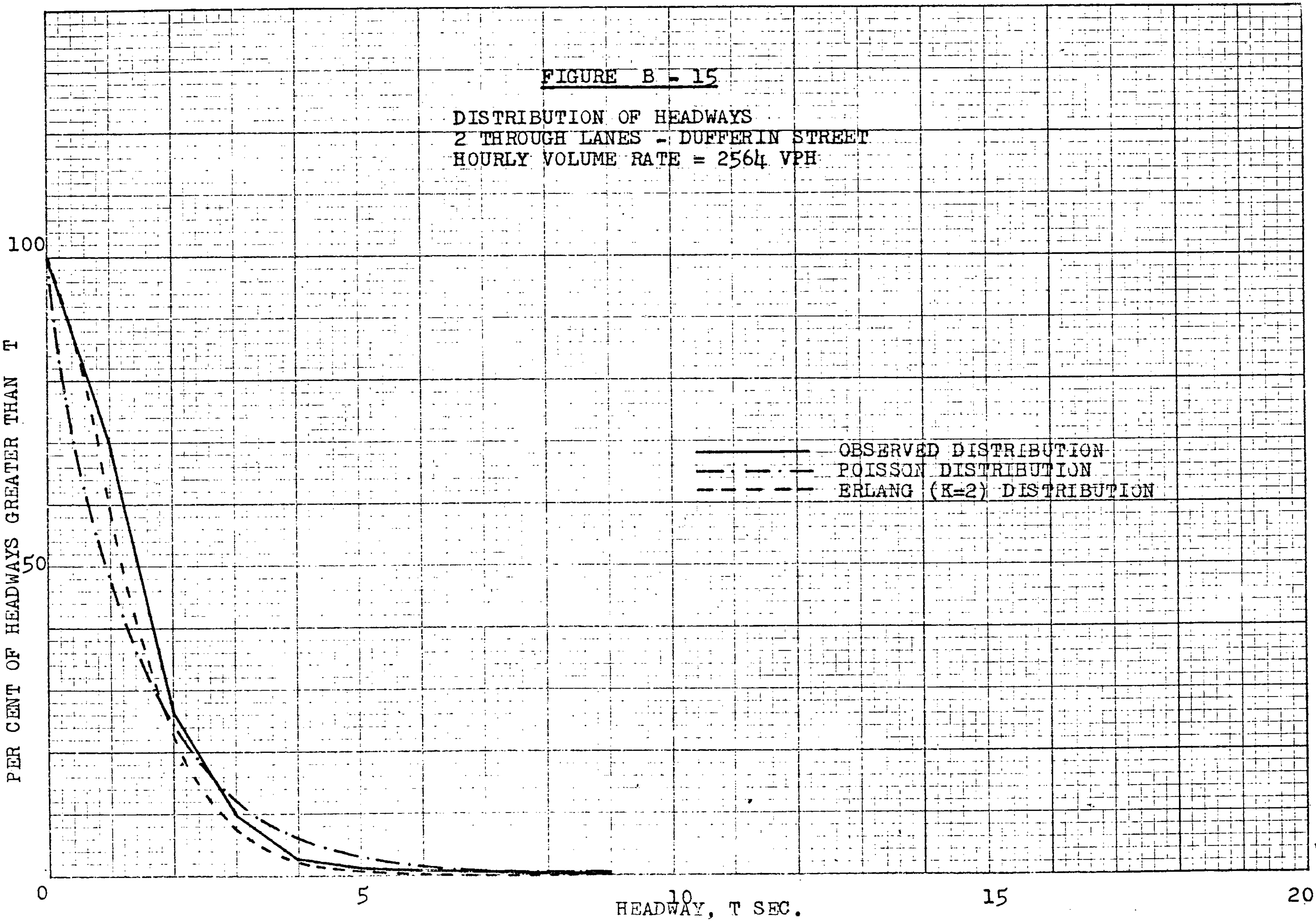


FIGURE B - 15

DISTRIBUTION OF HEADWAYS
2 THROUGH LANES - DUFFERIN STREET
HOURLY VOLUME RATE = 2564 VPH

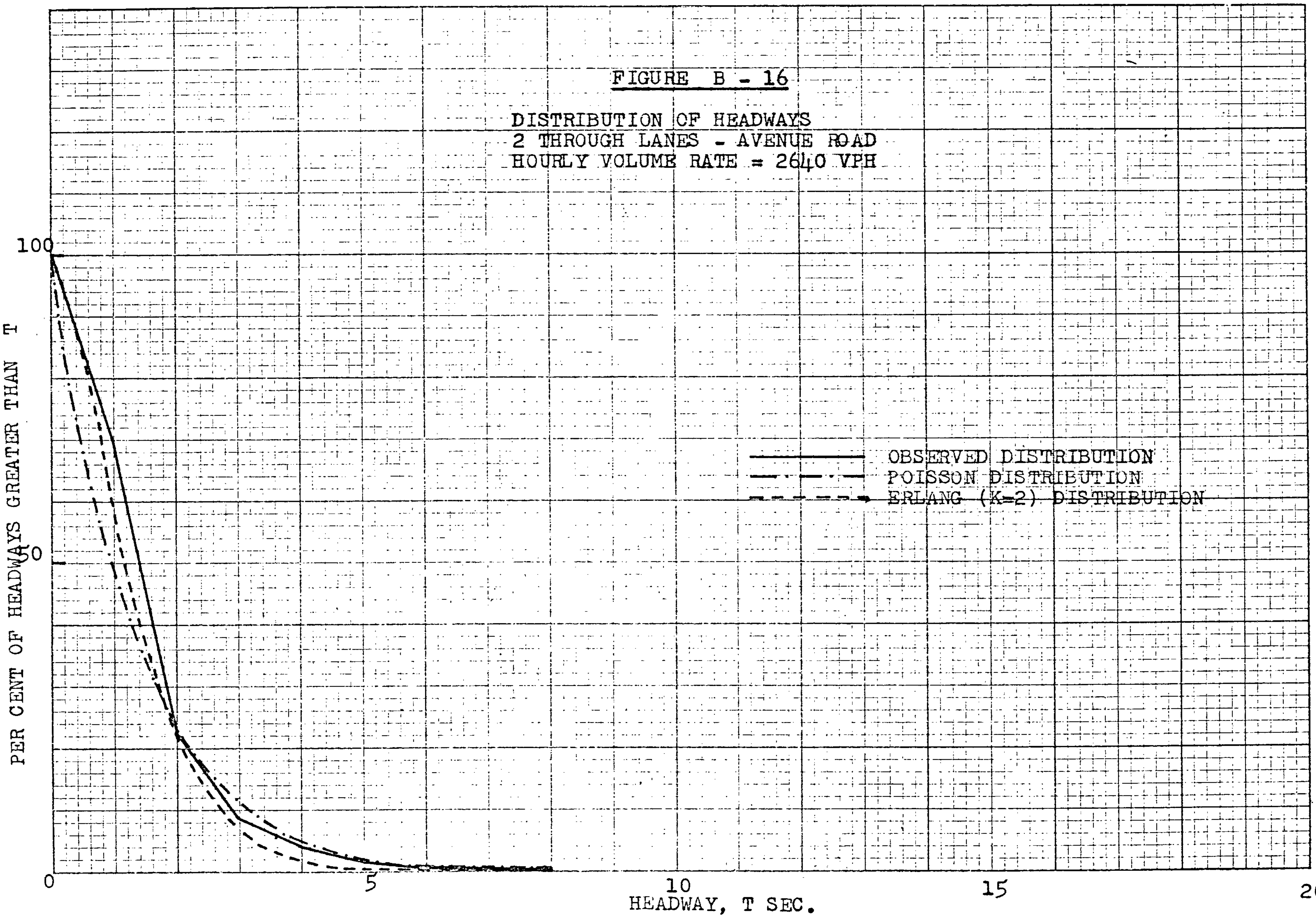


FIGURE B - 16
DISTRIBUTION OF HEADWAYS
2 THROUGH LANES - AVENUE ROAD
HOURLY VOLUME RATE = 2640 VPH
OBSERVED DISTRIBUTION
POISSON DISTRIBUTION
ERLANG (K=2) DISTRIBUTION
PER CENT OF HEADWAYS GREATER THAN T
100
50
0
5
10
15
20
HEADWAY, T SEC.

APPENDIX "C"

Curves (plotted on semi-log paper) of observed and theoretical distributions of Headways at all study locations.

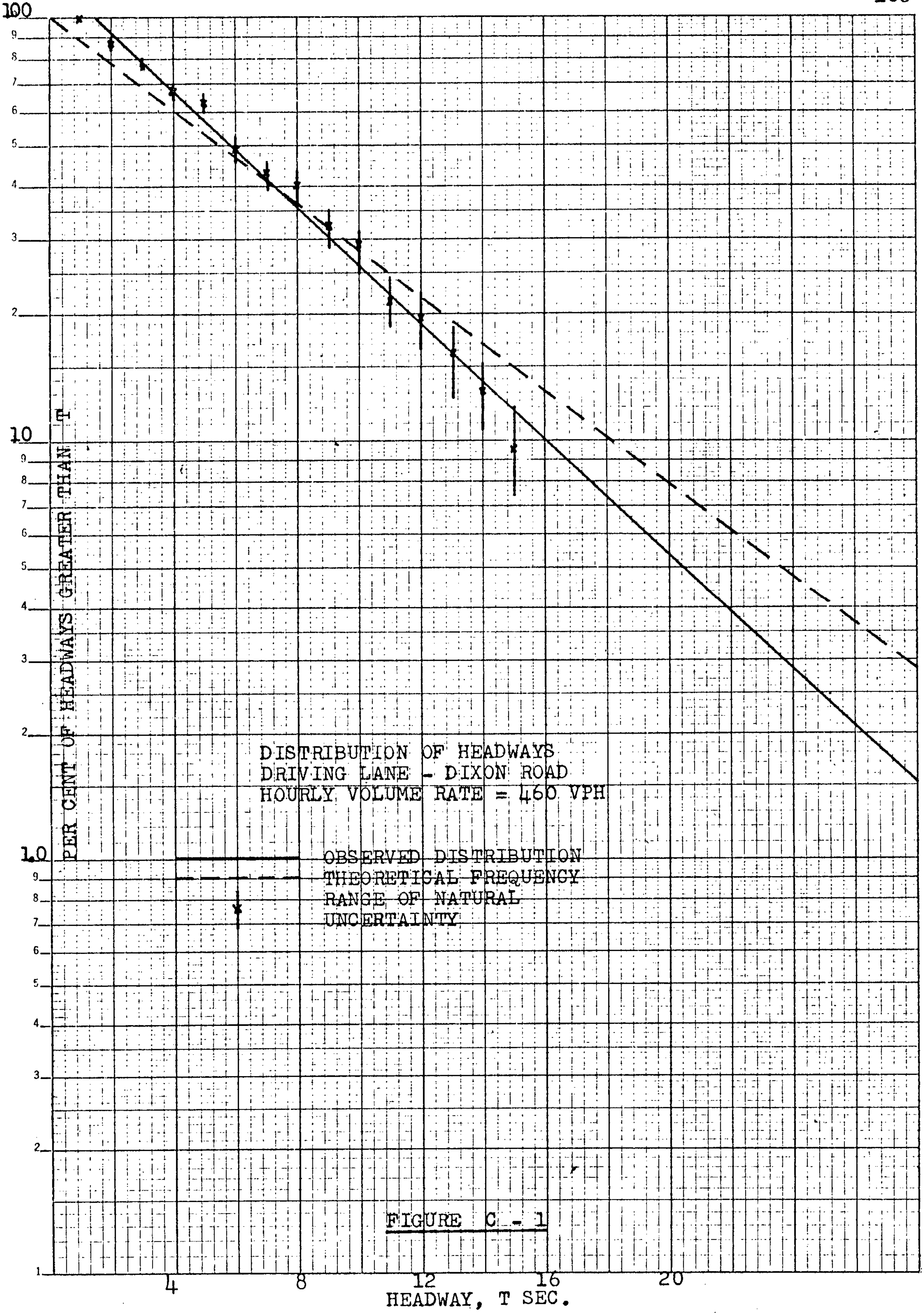

100
10
1.0
1
PER CENT OF HEADWAYS GREATER THAN T
DISTRIBUTION OF HEADWAYS
DRIVING LANE - DIXON ROAD
HOURLY VOLUME RATE = 460 VPH
OBSERVED DISTRIBUTION
THEORETICAL FREQUENCY
RANGE OF NATURAL UNCERTAINTY
4
8
12
16
20
HEADWAY, T SEC.

FIGURE C - 1

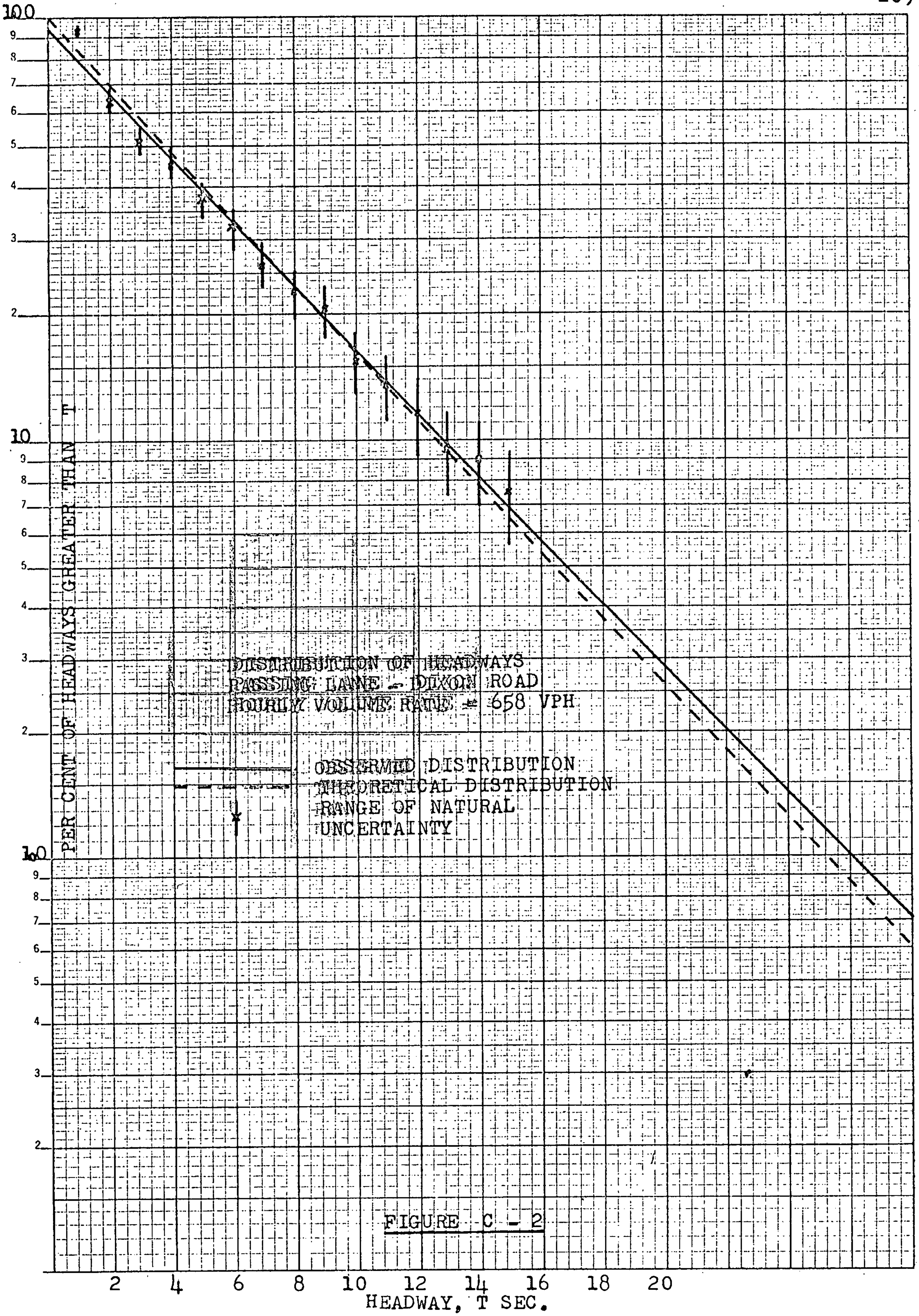

DISTRIBUTION OF HEADWAYS
PASSING LANE - DIXON ROAD
HOURLY VOLUME RATE = 658 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL
UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
2
4
6
8
10
12
14
16
18
20

FIGURE C - 2

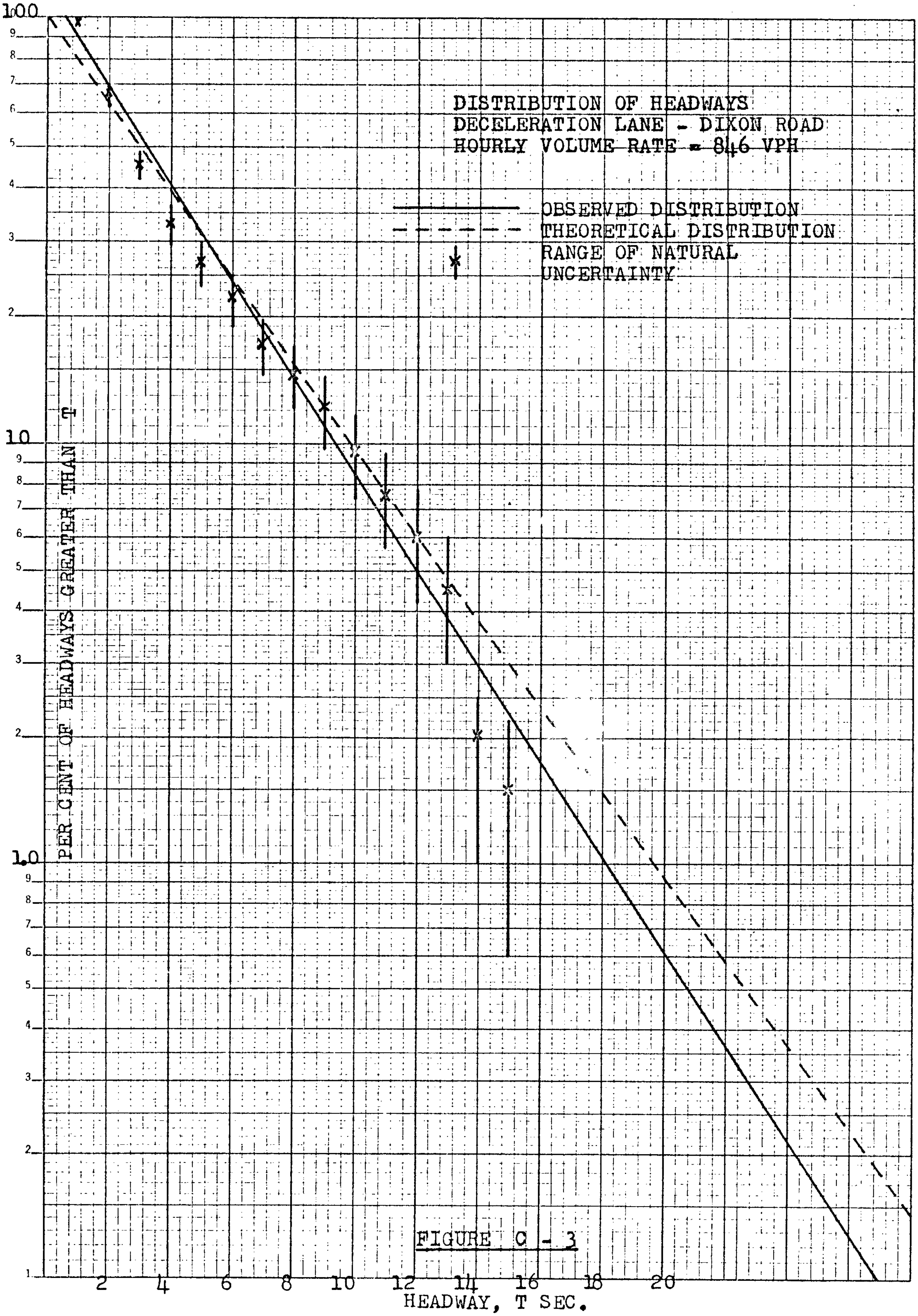

DISTRIBUTION OF HEADWAYS
DECELERATION LANE - DIXON ROAD
HOURLY VOLUME RATE = 846 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
100
10
1.0
1
2
4
6
8
10
12
14
16
18
20
HEADWAY, T SEC.

FIGURE C - 3

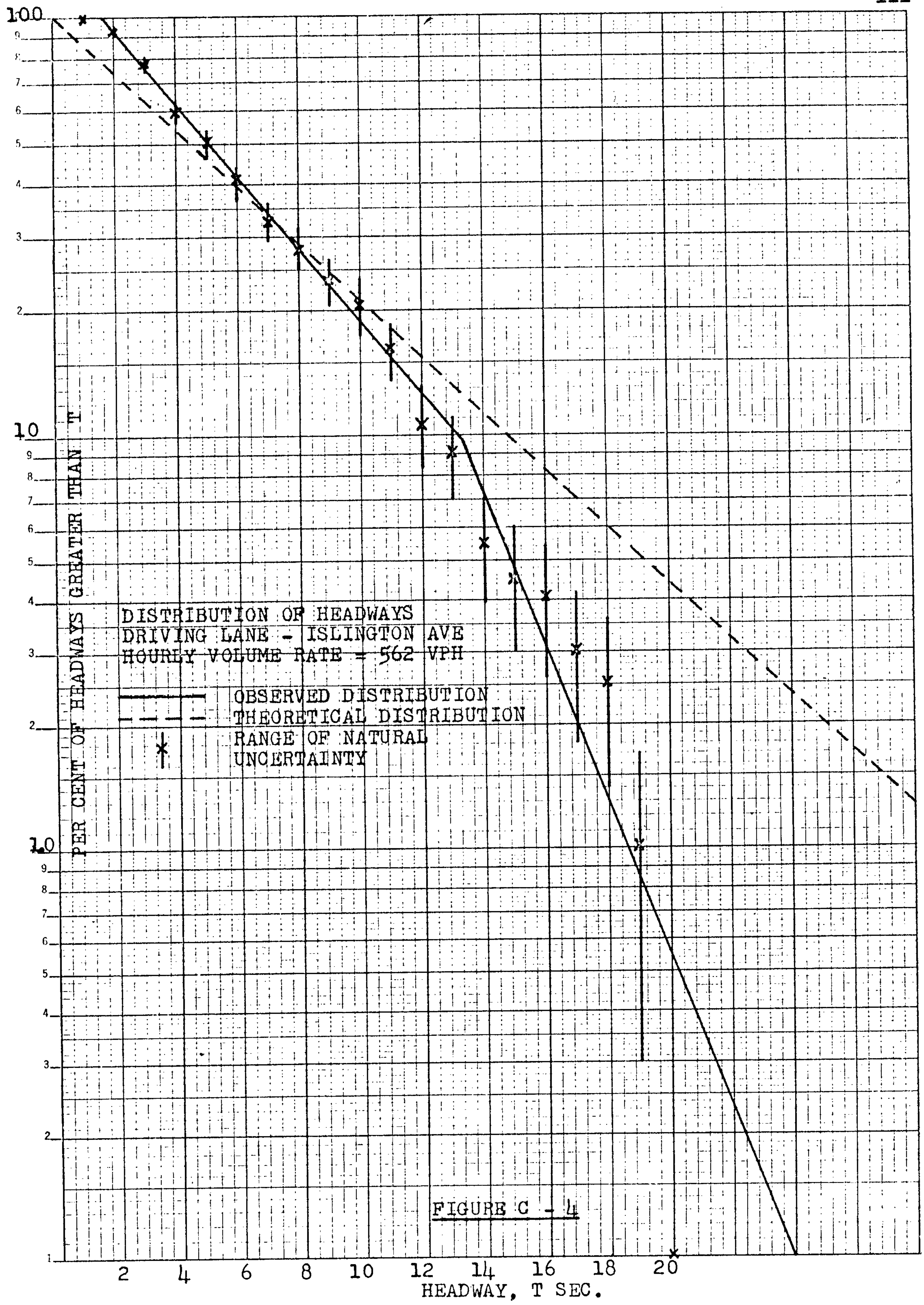

PER CENT OF HEADWAYS GREATER THAN T
DISTRIBUTION OF HEADWAYS
DRIVING LANE - ISLINGTON AVE
HOURLY VOLUME RATE = 562 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
HEADWAY, T SEC.

FIGURE C - 4

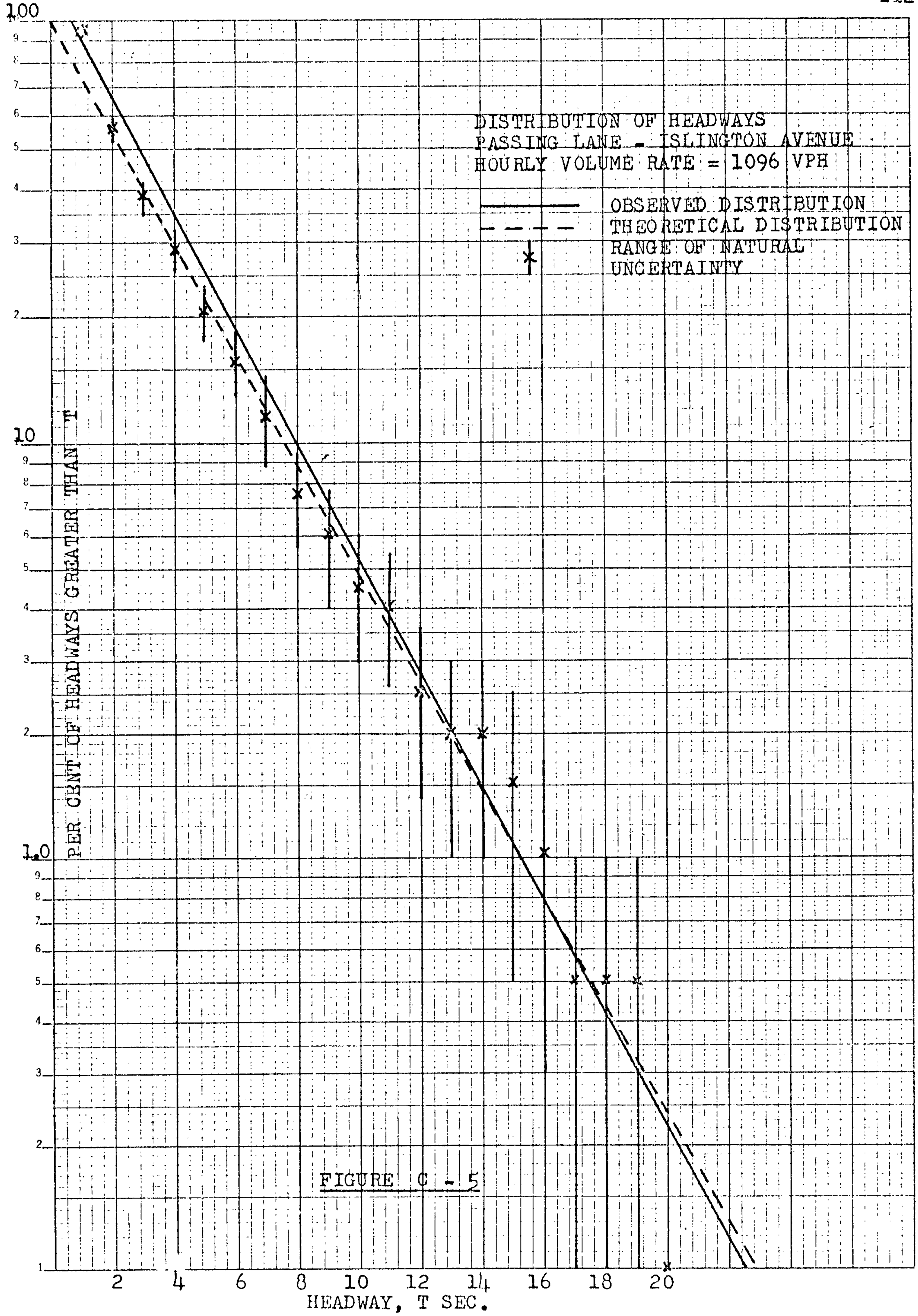
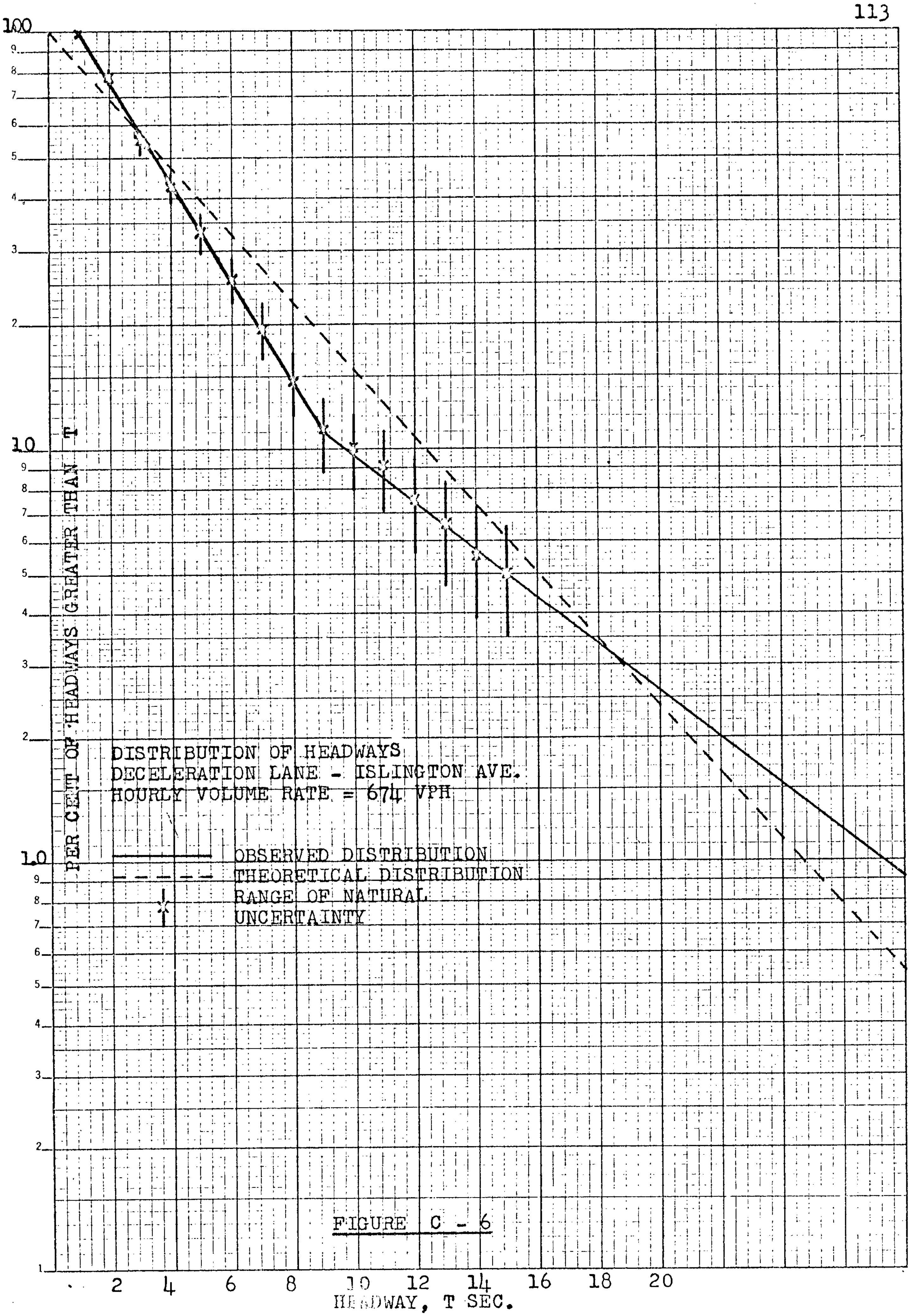

DISTRIBUTION OF HEADWAYS
PASSING LANE - ISLINGTON AVENUE
HOURLY VOLUME RATE = 1096 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
100
10
1.0
2
4
6
8
10
12
14
16
18
20
HEADWAY, T SEC.

FIGURE C - 5



DISTRIBUTION OF HEADWAYS
DECELERATION LANE - ISLINGTON AVE.
HOURLY VOLUME RATE = 674 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL
UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
2
4
6
8
10
12
14
16
18
20

FIGURE C - 6

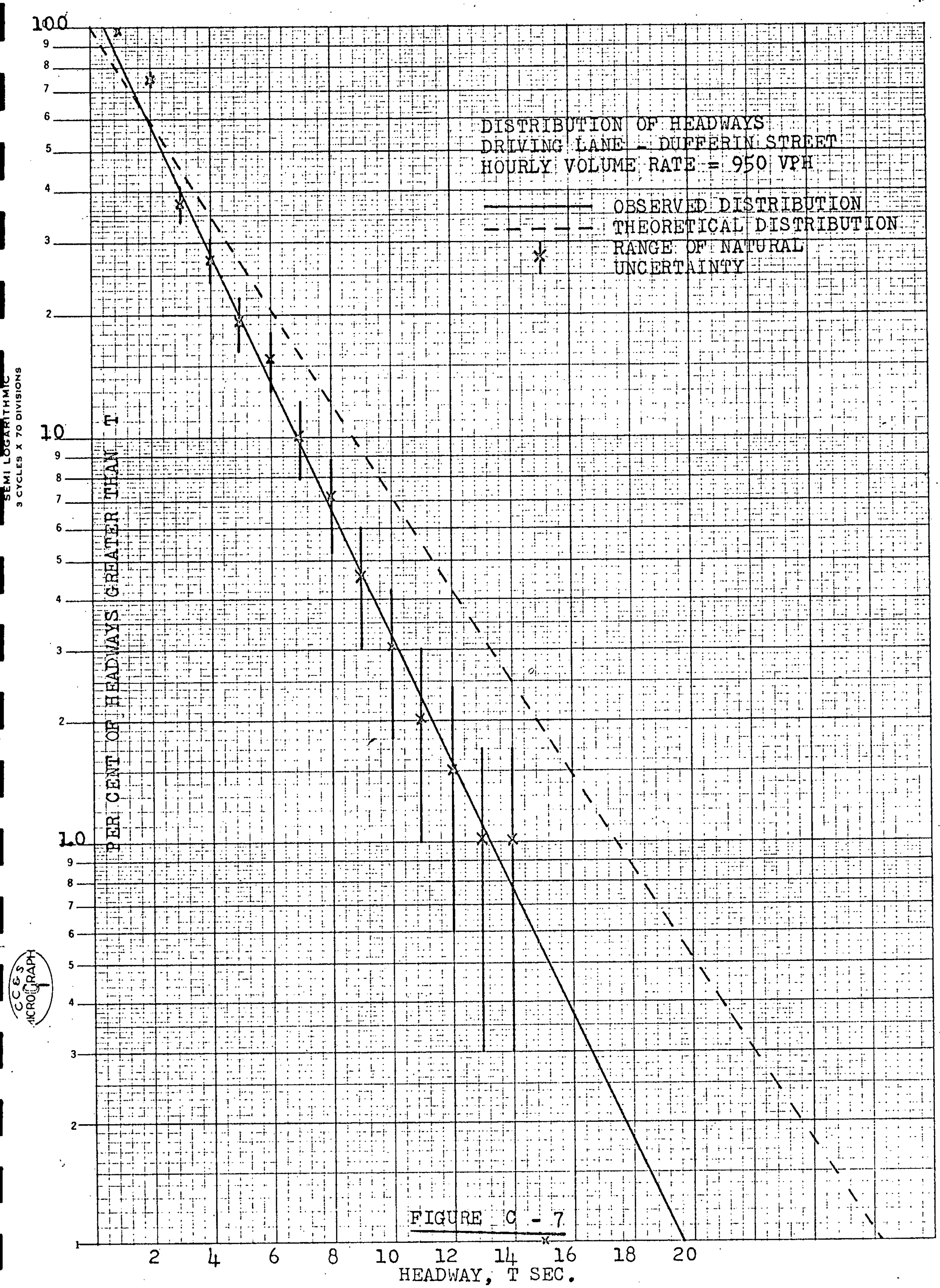

DISTRIBUTION OF HEADWAYS
DRIVING LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 950 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
2
4
6
8
10
12
14
16
18
20
SEMI LOGARITHMIC
3 CYCLES X 70 DIVISIONS

FIGURE C - 7

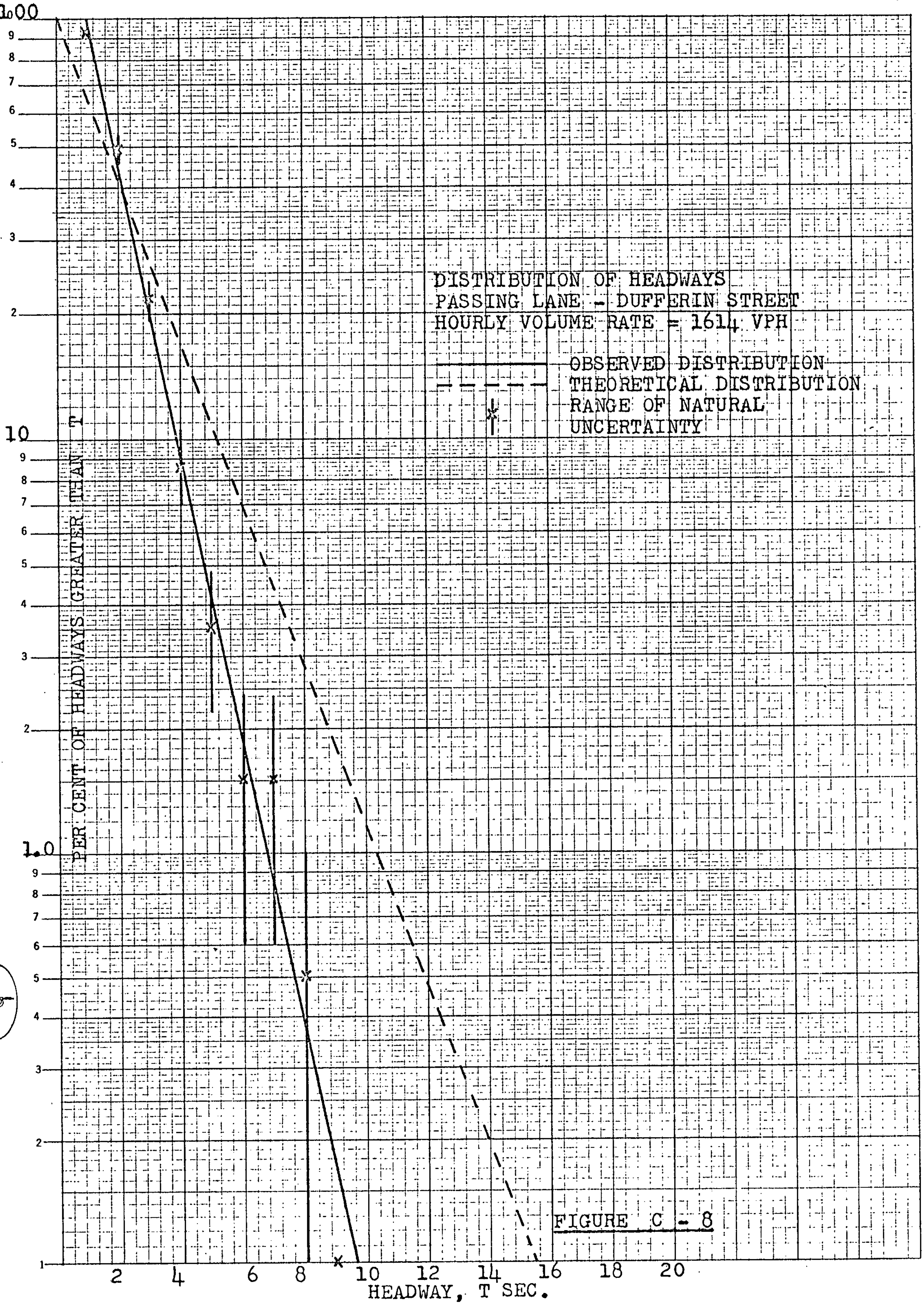

DISTRIBUTION OF HEADWAYS
PASSING LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 1614 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL
UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
1
2
4
6
8
10
12
14
16
18
20

FIGURE C - 8

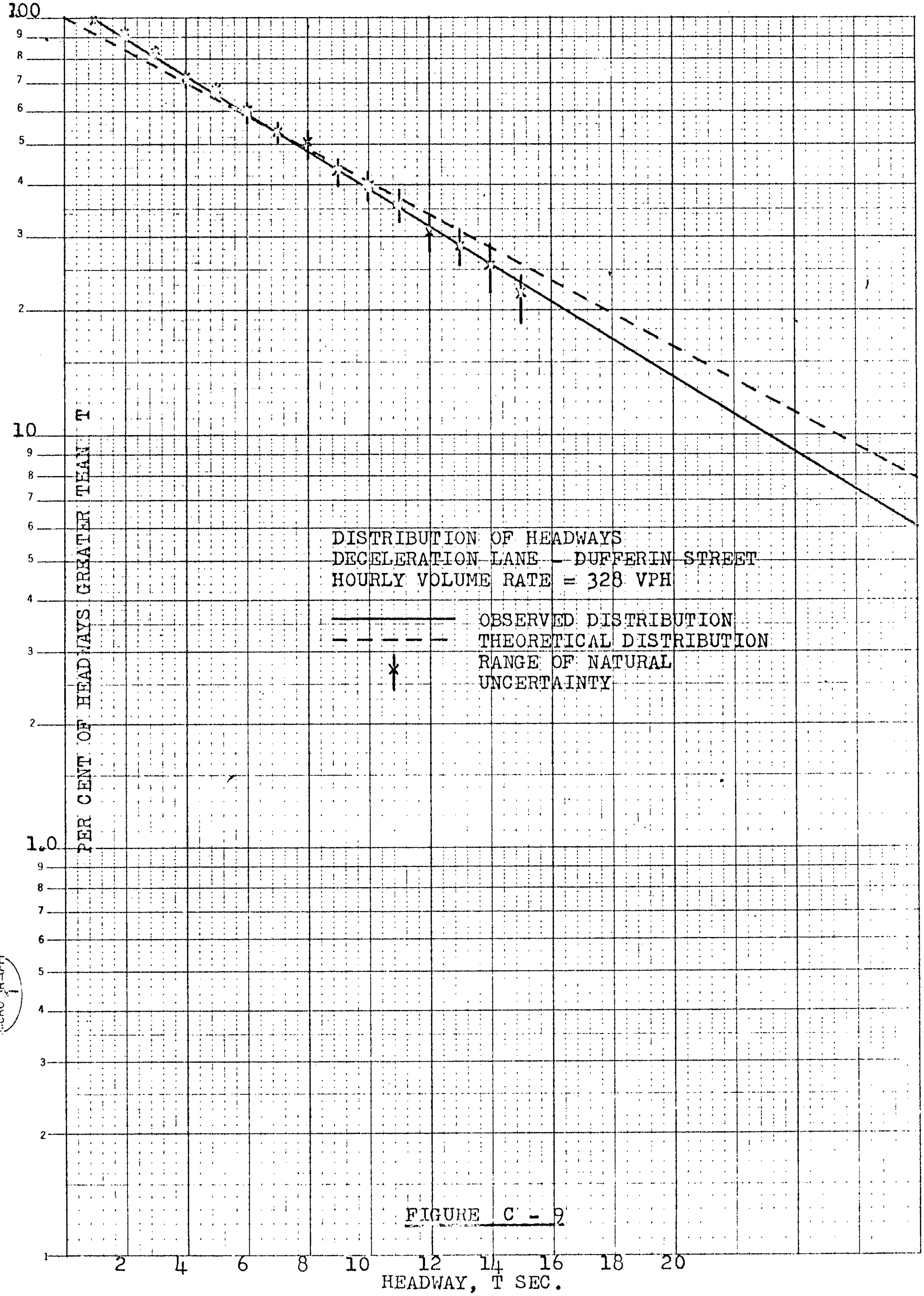

DISTRIBUTION OF HEADWAYS
DECELERATION LANE - DUFFERIN STREET
HOURLY VOLUME RATE = 328 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
2
4
6
8
10
12
14
16
18
20

FIGURE C - 9

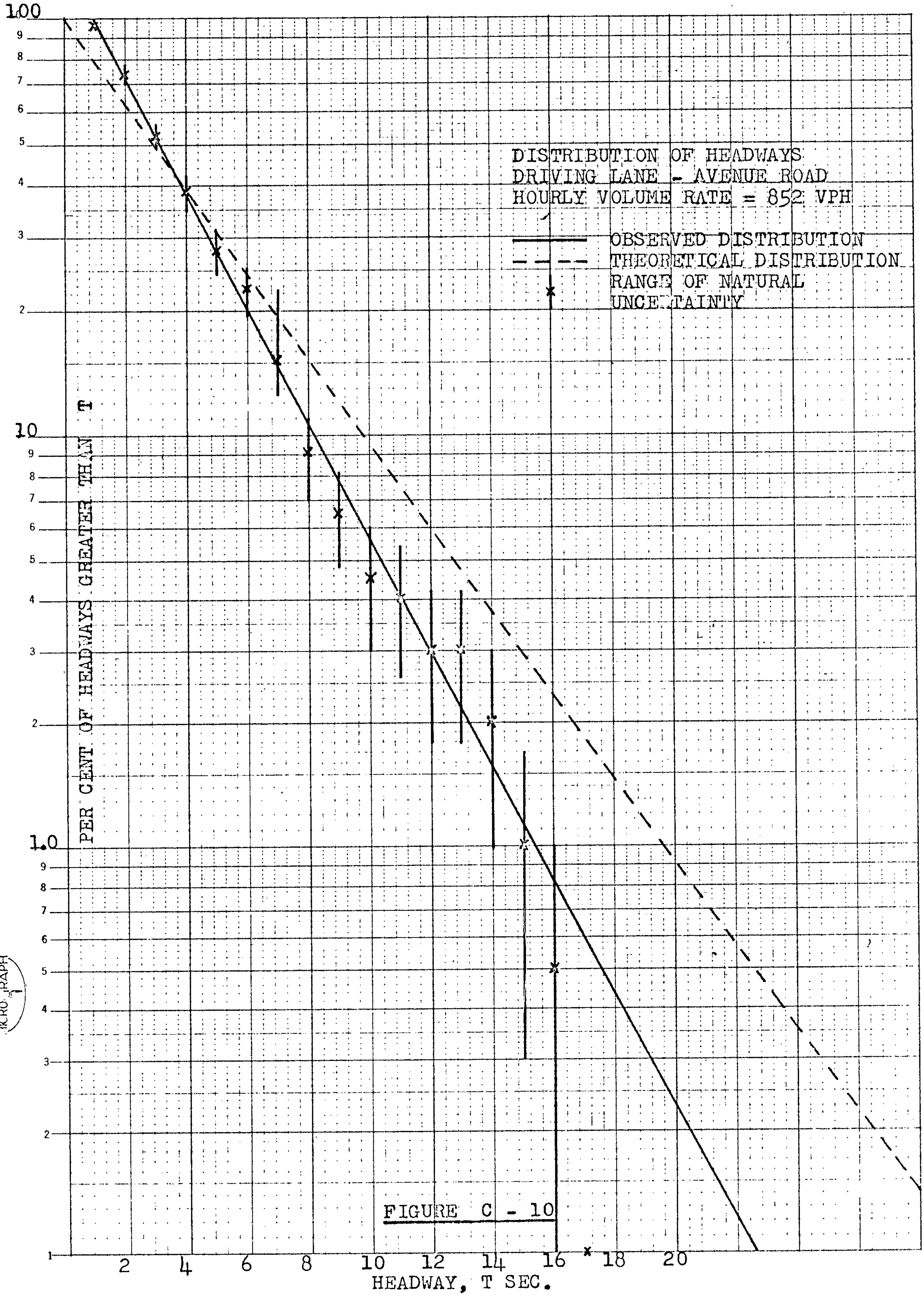
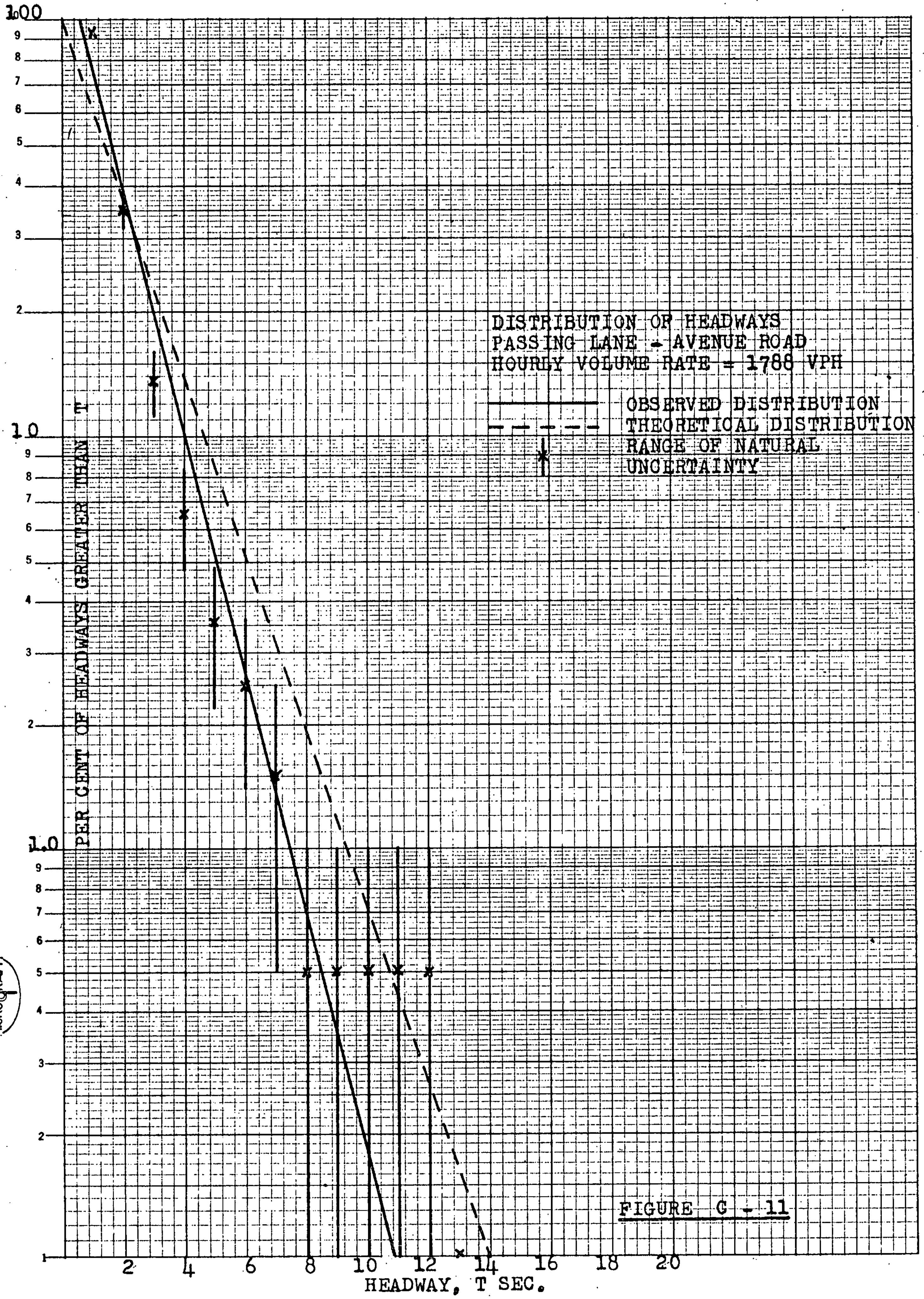

DISTRIBUTION OF HEADWAYS
DRIVING LANE - AVENUE ROAD
HOURLY VOLUME RATE = 852 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
2
4
6
8
10
12
14
16
18
20

FIGURE C - 10



DISTRIBUTION OF HEADWAYS
PASSING LANE - AVENUE ROAD
HOURLY VOLUME RATE = 1788 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
2
4
6
8
10
12
14
16
18
20

FIGURE G - 11

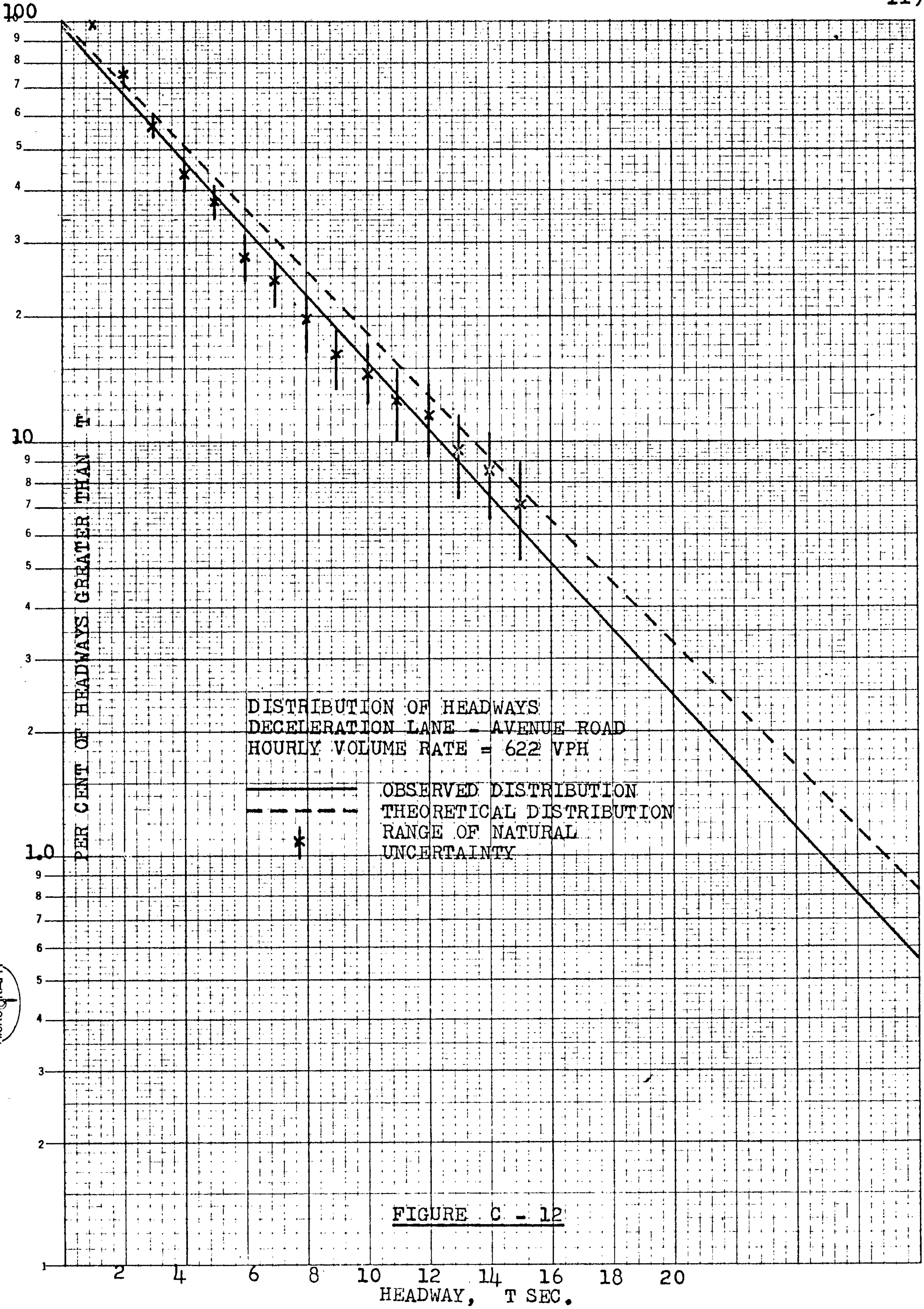

PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
DISTRIBUTION OF HEADWAYS
DECELERATION LANE - AVENUE ROAD
HOURLY VOLUME RATE = 622 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
100
10
1.0
2
4
6
8
10
12
14
16
18
20

FIGURE C - 12

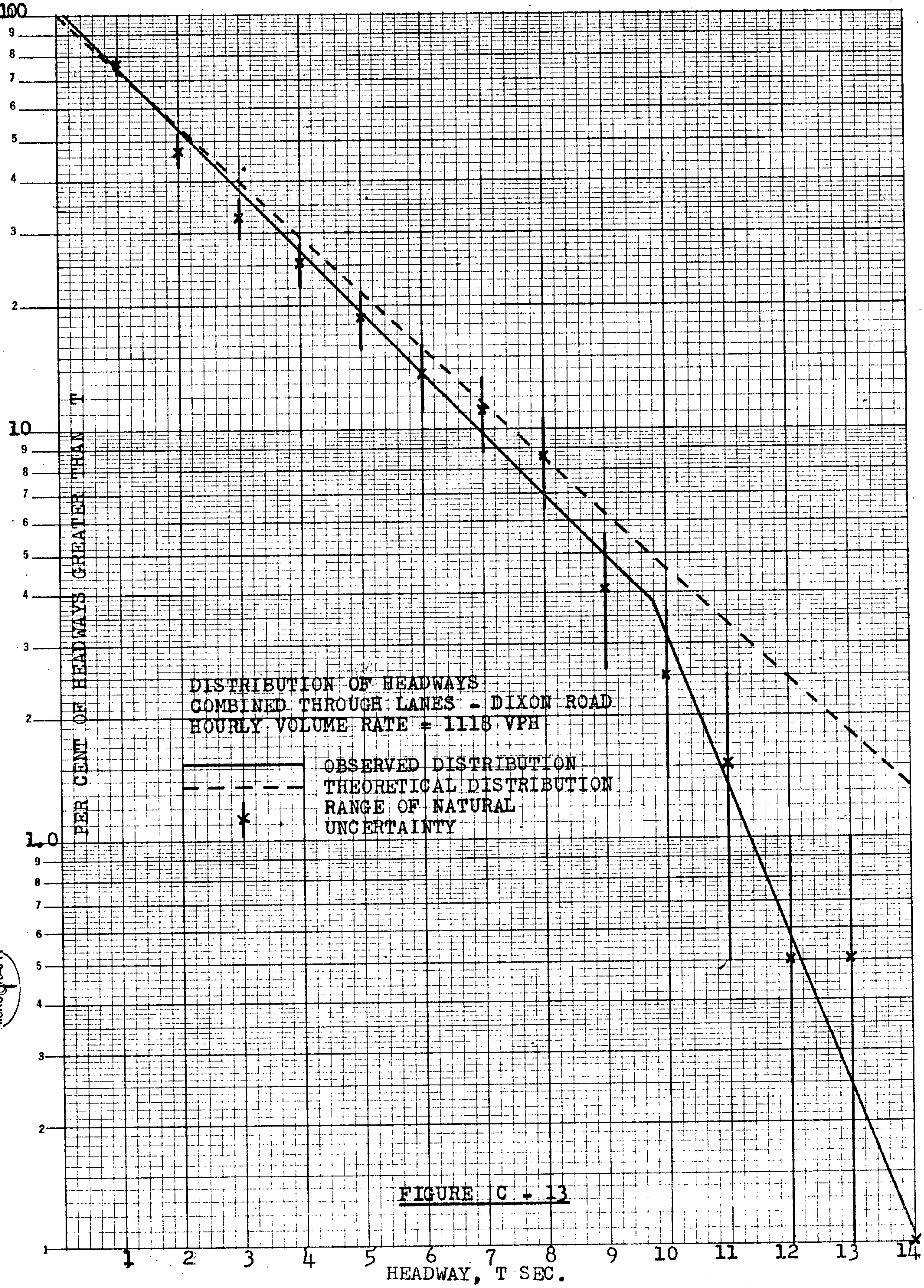

PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
DISTRIBUTION OF HEADWAYS
COMBINED THROUGH LANES - DIXON ROAD
HOURLY VOLUME RATE = 1118 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL
UNCERTAINTY
100
10
1.0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIGURE C - 13

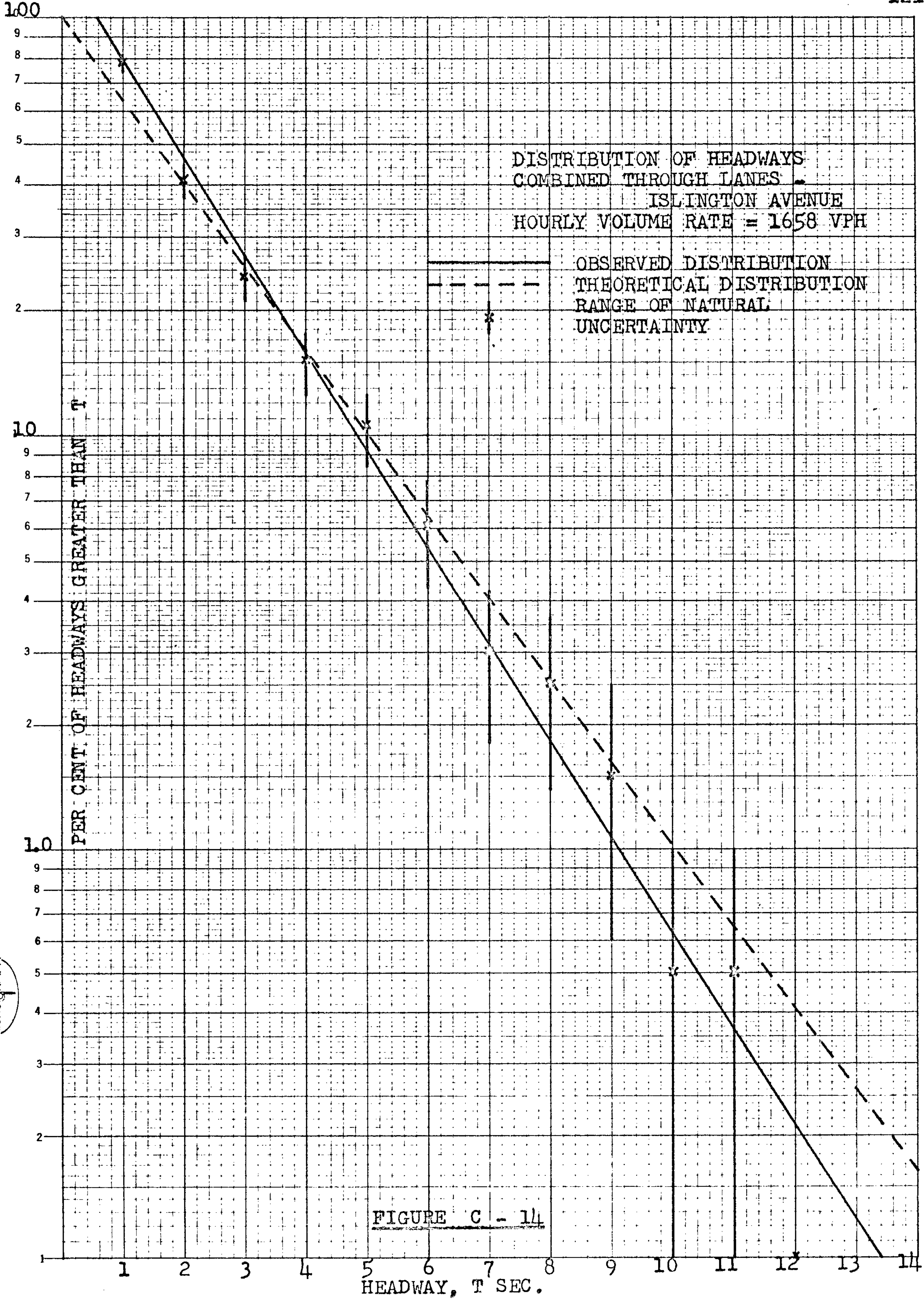

DISTRIBUTION OF HEADWAYS
COMBINED THROUGH LANES -
ISLINGTON AVENUE
HOURLY VOLUME RATE = 1658 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL
UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIGURE C - 14

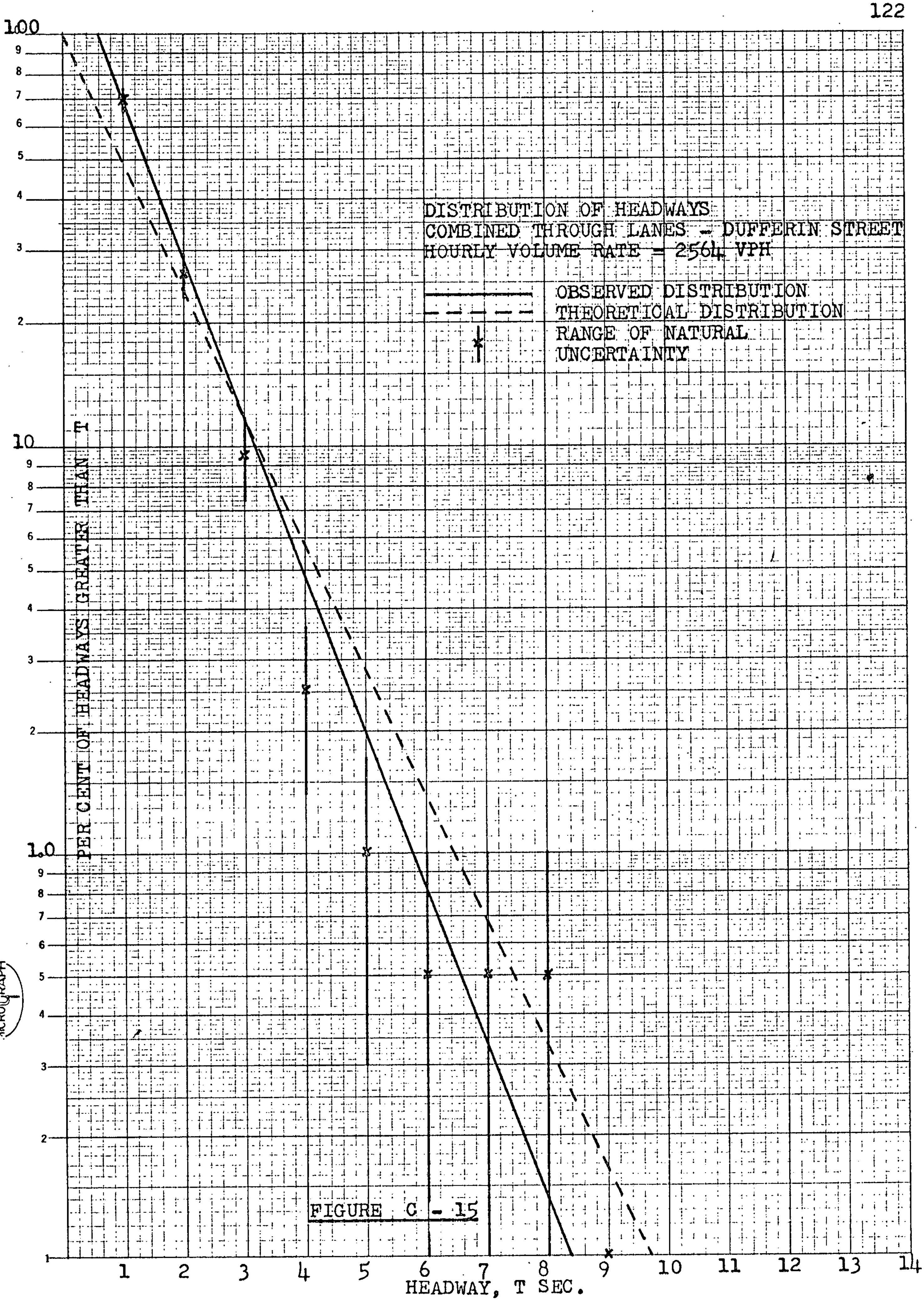

DISTRIBUTION OF HEADWAYS
COMBINED THROUGH LANES - DUFFERIN STREET
HOURLY VOLUME RATE = 2564 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIGURE C - 15

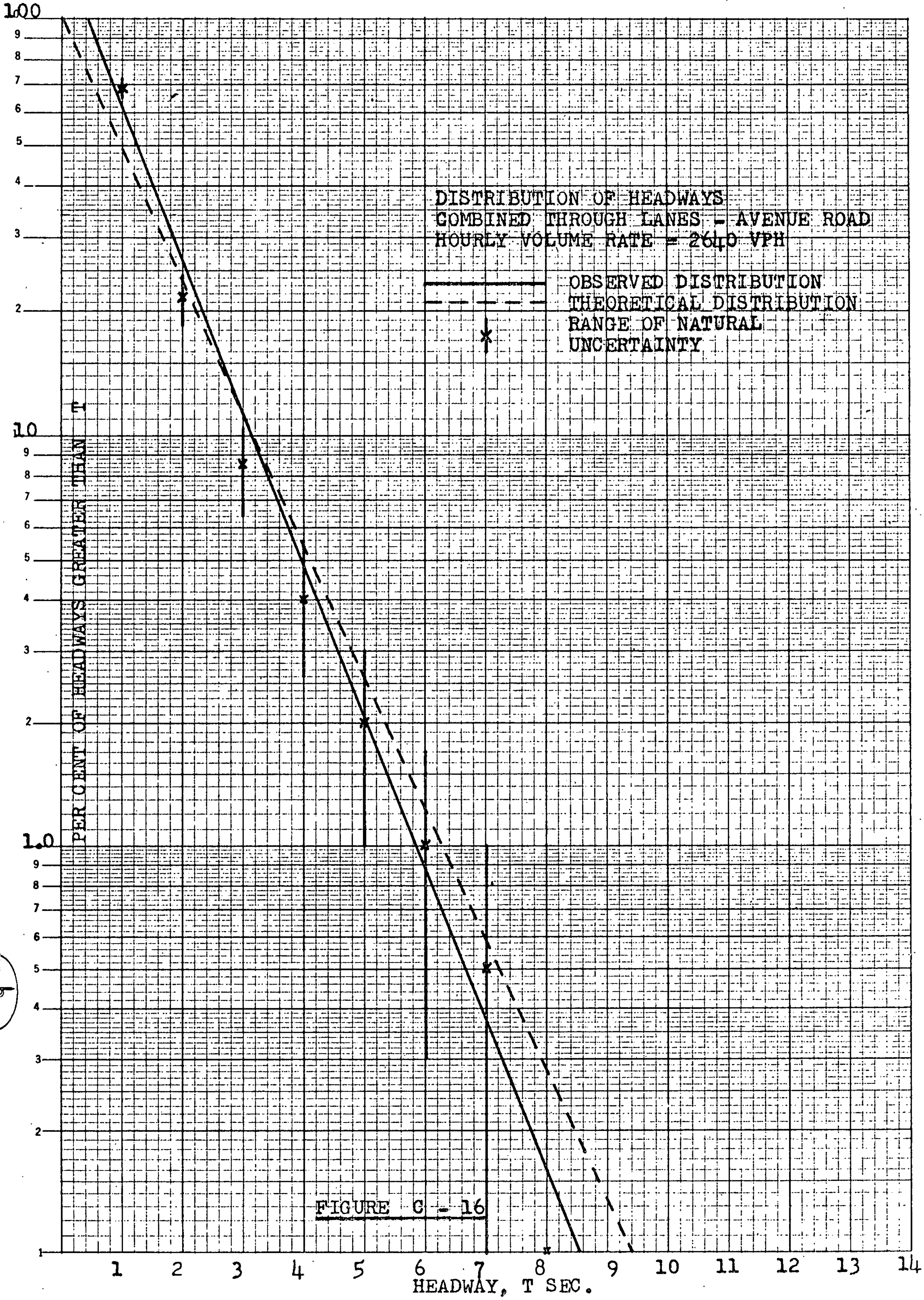

DISTRIBUTION OF HEADWAYS
COMBINED THROUGH LANES - AVENUE ROAD
HOURLY VOLUME RATE = 2640 VPH
OBSERVED DISTRIBUTION
THEORETICAL DISTRIBUTION
RANGE OF NATURAL UNCERTAINTY
PER CENT OF HEADWAYS GREATER THAN T
HEADWAY, T SEC.
100
10
1.0
1
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIGURE C - 16

APPENDIX "D"

Tables of Chi-Square Tests when fitting observed data to Poisson and Erlang (K=2) distributions.

TABLE D-1

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
DRIVING LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.1193	24	23	529	22.04
1.0-1.9	24	0.1050	21	3	9	0.43
2.0-2.9	19	0.0925	19	0	0	0.00
3.0-3.9	22	0.0815	16	6	36	2.25
4.0-4.9	18	0.0718	14	4	16	1.14
5.0-5.9	8	0.0632	13	5	25	1.92
6.0-6.9	13	0.0555	11	2	4	0.36
7.0-7.9	15	0.0491	10	5	25	2.50
8.0-8.9	16	0.0431	9	7	49	5.44
9.0-9.9	8	0.0381	8	0	0	0.00
10.0-10.9	13	0.0335	7	6	36	5.14
11.0-11.9	4	0.0295	6	2	4	0.67
12.0-12.9	7	0.0259	5	2	4	0.80
13.0-13.9	6	0.0229	5	1	1	0.20
14.0-14.9	7 } 26	0.0202	4 } 34	8	64	1.88
≥15	19	0.1488	30			
	N=200					Σ=44.77

$X^2_{(13,0.01)} = 27.69$

$44.77 > 27.69$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-2

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.0273	5	4	16	3.20
1.0-1.9	24	0.0653	13	11	121	9.31
2.0-2.9	19	0.0851	17	2	4	0.24
3.0-3.9	22	0.0925	19	3	9	0.47
4.0-4.9	18	0.0924	18	0	0	0.00
5.0-5.9	8	0.0877	18	10	100	5.56
6.0-6.9	13	0.0802	16	3	9	0.56
7.0-7.9	15	0.0720	14	1	1	0.07
8.0-8.9	16	0.0633	13	3	9	0.69
9.0-9.9	8	0.0549	11	3	9	0.82
10.0-10.9	13	0.0471	9	4	16	1.78
11.0-11.9	4	0.0403	8	4	16	2.00
12.0-12.9	7	0.0336	7	0	0	0.00
13.0-13.9	6	0.0285	6	0	0	0.00
14.0-14.9	7	0.0235	5	2	4	0.80
≥15	19	0.1063	21	2	4	0.19
	N=200					Σ =25.69

$X^2_{(14,0.01)} = 29.14$

25.69 < 29.14 ∴ Erlang fits at the 1% significance level.

TABLE D-3

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: PASSING LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	15	0.1664	33	18	324	9.82
1.0-1.9	56	0.1387	28	28	784	28.00
2.0-2.9	27	0.1156	23	4	16	0.70
3.0-3.9	11	0.0964	19	8	64	3.37
4.0-4.9	16	0.0804	16	0	0	0.00
5.0-5.9	11	0.0670	13	2	4	0.31
6.0-6.9	12	0.0558	11	1	1	0.09
7.0-7.9	7	0.0465	9	2	4	0.44
8.0-8.9	4	0.0388	8	4	16	2.00
9.0-9.9	10	0.0324	7	3	9	1.29
10.0-10.9	4	0.0269	5	1	1	0.20
11.0-11.9	4	0.0225	5	1	1	0.20
12.0-12.9	4 } 8	0.0187	4 } 10	=2	4	0.40
13.0-13.9	1	0.0157	3			
14.0-14.9	3	0.0130	3			
≥15	15	0.0652	13	2	4	0.31
	N=200					Σ =47.13

$X^2_{(12,0.01)} = 26.22$

$47.13 > 26.22$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-4

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
PASSING LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	15	0.0522	10	5	25	2.50
1.0-1.9	56	0.1133	23	33	1089	47.35
2.0-2.9	27	0.1326	27	0	0	0.00
3.0-3.9	11	0.1292	26	15	225	8.65
4.0-4.9	16	0.1159	23	7	49	2.13
5.0-5.9	11	0.0983	20	9	81	4.05
6.0-6.9	12	0.0810	16	4	16	1.00
7.0-7.9	7	0.0647	13	6	36	2.77
8.0-8.9	4	0.0490	10	6	36	3.60
9.0-9.9	10	0.0418	8	2	4	0.50
10.0-10.9	4	0.0309	6	2	4	0.67
11.0-11.9	4	0.0229	5	1	1	0.20
12.0-12.9	4 }	0.0178	4 }			
13.0-13.9	1 } 8	0.0132	3 } 9	1	1	0.11
14.0-14.9	3 }	0.0094	2 }			
≥15	15	0.0278	6	9	81	13.50
	N=200					Σ=87.03

$X^2_{(12,0.01)} = 26.22$

$87.03 > 26.22$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-5

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: DECELERATION LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	5	0.2094	42	37	1369	32.60
1.0-1.9	63	0.1656	33	30	900	27.27
2.0-2.9	42	0.1309	26	16	256	9.85
3.0-3.9	25	0.1035	21	4	16	0.76
4.0-4.9	12	0.0818	16	4	16	1.00
5.0-5.9	9	0.0647	13	4	16	1.23
6.0-6.9	10	0.0511	10	0	0	0.00
7.0-7.9	5	0.0404	8	3	9	1.13
8.0-8.9	5	0.0320	7	2	4	0.57
9.0-9.9	5	0.0252	5	0	0	0.00
10.0-10.9	4	0.0200	4			
11.0-11.9	3	0.0158	3			
12.0-12.9	3 } 16	0.0120	2 } 13	3	9	0.69
13.0-13.9	5	0.0103	2			
14.0-14.9	1	0.0079	2			
≥15	3	0.0294	6	3	9	1.50
	N=200					Σ=76.60

$X^2_{(10,0.01)} = 23.21$

$76.60 > 23.21$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-6

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	5	0.0812	16	11	121	7.56
1.0-1.9	63	0.1610	32	31	961	30.03
2.0-2.9	42	0.1695	34	8	64	1.88
3.0-3.9	25	0.1488	30	5	25	0.84
4.0-4.9	12	0.1199	24	12	144	6.00
5.0-5.9	9	0.0919	18	9	81	4.50
6.0-6.9	10	0.0677	14	4	16	1.14
7.0-7.9	5	0.0491	10	5	25	2.50
8.0-8.9	5	0.0345	7	2	4	0.57
9.0-9.9	5	0.0245	5	0	0	0.00
10.0-10.9	4	0.0167	3			
11.0-11.9	3	0.0113	2			
12.0-12.9	3 } 19	0.0083	2 } 11	8	64	5.82
13.0-13.9	5	0.0050	1			
14.0-14.9	1	0.0034	1			
≥15	3	0.0072	2			
	N=200					Σ=60.84

$X^2_{(9,0.01)} = 21.67$

$60.84 > 21.67$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-7

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.1444	29	28	784	27.03
1.0-1.9	15	0.1236	25	10	100	4.00
2.0-2.9	30	0.1057	22	8	64	2.91
3.0-3.9	36	0.0905	18	18	324	18.00
4.0-4.9	18	0.0774	16	2	4	0.25
5.0-5.9	20	0.0662	13	7	49	3.77
6.0-6.9	15	0.0567	11	4	16	0.36
7.0-7.9	9	0.0484	10	1	1	0.10
8.0-8.9	9	0.0415	8	1	1	0.13
9.0-9.9	6	0.0355	7	1	1	0.14
10.0-10.9	9	0.0303	6	3	9	1.50
11.0-11.9	11	0.0260	5	6	36	7.20
12.0-12.9	3	0.0222	4			
13.0-13.9	7	0.0190	4			
14.0-14.9	2	0.0163	3			
15.0-15.9	1 } 21	0.0139	3 } 21	0	0	0.00
16.0-16.9	2	0.0119	2			
17.0-17.9	1	0.0102	2			
18.0-18.9	3	0.0087	2			
19.0-19.9	2	0.0074	1			
	N=200					Σ=65.39

$X^2_{(11,0.01)} = 24.73$

$65.39 > 24.73$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-8

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
DRIVING LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.0397	8	7	49	6.13
1.0-1.9	15	0.0902	18	3	9	0.50
2.0-2.9	30	0.1108	22	8	64	2.91
3.0-3.9	36	0.1139	23	13	169	7.35
4.0-4.9	18	0.1075	22	4	16	0.73
5.0-5.9	20	0.0962	19	1	1	0.05
6.0-6.9	15	0.0832	17	2	4	0.24
7.0-7.9	9	0.0704	14	5	25	1.79
8.0-8.9	9	0.0585	11	2	4	0.36
9.0-9.9	6	0.0475	9	3	9	1.00
10.0-10.9	9	0.0385	7	2	4	0.57
11.0-11.9	11	0.0316	6	5	25	4.17
12.0-12.9	3	0.0245	4			
13.0-13.9	7	0.0193	3			
14.0-14.9	2	0.0154	3 } 17	4	16	0.94
15.0-15.9	1 } 21	0.0121	2			
16.0-16.9	2	0.0092	2			
17.0-17.9	1	0.0077	1			
18.0-18.9	3	0.0051	1			
19.0-19.9	2	0.0049	1			
	N =200					Σ =26.74

$X^2_{(11,0.01)} = 24.73$

$26.74 > 24.73$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-9

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
PASSING LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	12	0.2629	53	41	1681	31.72
1.0-1.9	77	0.1937	39	38	1444	37.03
2.0-2.9	35	0.1429	29	6	36	1.24
3.0-3.9	19	0.1053	21	2	4	0.19
4.0-4.9	16	0.0776	15	1	1	0.07
5.0-5.9	10	0.0572	11	1	1	0.09
6.0-6.9	8	0.0422	8	0	0	0.00
7.0-7.9	8	0.0310	6	2	4	0.67
8.0-8.9	3	0.0230	5	2	4	0.80
9.0-9.9	3	0.0168	3			
10.0-10.9	1	0.0127	3			
11.0-11.9	3	0.0090	2			
12.0-12.9	1	0.0068	1			
13.0-13.9	0 } 12	0.0049	1 } 12	0	0	0.00
14.0-14.9	1	0.0037	1			
15.0-15.9	1	0.0027	1			
16.0-16.9	1	0.0020	0			
17.0-17.9	0	0.0015	0			
18.0-18.9	0	0.0011	0			
19.0-19.9	1	0.0008	0			
	N=200					$\Sigma=71.81$

$X^2_{(8,0.01)} = 20.09$

$71.81 > 20.09$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-10

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
PASSING LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	12	0.1251	25	13	169	6.76
1.0-1.9	77	0.2196	44	33	1089	24.75
2.0-2.9	35	0.2013	40	5	25	0.63
3.0-3.9	19	0.1540	31	12	144	4.65
4.0-4.9	16	0.1080	21	5	25	1.19
5.0-5.9	10	0.0722	14	4	16	1.14
6.0-6.9	8	0.0460	9	1	1	0.14
7.0-7.9	8	0.0291	6	2	4	0.67
8.0-8.9	3	0.0181	4			
9.0-9.9	3	0.0110	2			
10.0-10.9	1	0.0063	1			
11.0-11.9	3	0.0045	1			
12.0-12.9	1	0.0022	0			
13.0-13.9	0 }15	0.0017	0 }8	7	49	6.13
14.0-14.9	1	0.0009	0			
15.0-15.9	1	0.0004	0			
16.0-16.9	1	0.0003	0			
17.0-17.9	0	0.0001	0			
18.0-18.9	0	0.0001	0			
19.0-19.9	1	0.00003	0			
	N=200					Σ=46.06

$X^2(7,0.01) = 18.48$

$46.06 > 18.48$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-11

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.1706	34	33	1089	32.03
1.0-1.9	43	0.1414	28	15	225	8.04
2.0-2.9	47	0.1174	24	23	529	22.04
3.0-3.9	25	0.0973	19	6	36	1.89
4.0-4.9	18	0.0807	16	2	4	0.25
5.0-5.9	15	0.0670	13	2	4	0.31
6.0-6.9	12	0.0555	11	1	1	0.09
7.0-7.9	10	0.0461	9	1	1	0.11
8.0-8.9	7	0.0382	8	1	1	0.13
9.0-9.9	2	0.0317	6	4	16	2.67
10.0-10.9	2	0.0263	5	3	9	1.80
11.0-11.9	3 }	0.0218	4 }			
12.0-12.9	2 }	0.0181	4 } 13	5	25	1.92
13.0-13.9	2 } 8	0.0150	3 }			
14.0-14.9	1 }	0.0124	2 }			
≥15	10	0.0605	12	2	4	0.33
	N=200					Σ=71.61

$X^2_{(11,0.01)} = 24.73$

$71.61 > 24.73$ ∴ Poisson does not fit at the 1% significance level.

T

TABLE D-12

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: DECELERATION LANE - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.0547	11	10	100	9.09
1.0-1.9	43	0.1180	24	19	361	15.04
2.0-2.9	47	0.1364	27	20	400	14.81
3.0-3.9	25	0.1318	26	1	1	0.04
4.0-4.9	18	0.1168	23	5	25	1.09
5.0-5.9	15	0.0984	20	5	25	1.25
6.0-6.9	12	0.0801	16	4	16	1.00
7.0-7.9	10	0.0634	13	3	9	0.69
8.0-8.9	7	0.0498	10	3	9	0.90
9.0-9.9	2	0.0378	8	6	36	4.50
10.0-10.9	2	0.0294	6	4	16	2.67
11.0-11.9	3	0.0219	4			
12.0-12.9	2	0.0158	3			
13.0-13.9	2 } 8	0.0126	3 } 12	4	16	1.33
14.0-14.9	1	0.0086	2			
≥15	10	0.0245	5	5	25	5.00
	N=200					Σ=57.41

$X^2_{(11,0.01)} = 24.73$

$57.41 > 24.73$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-13

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)		PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)		(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	3		0.2313	46		43	1849	40.20
1.0-1.9	51		0.1777	36		15	225	6.25
2.0-2.9	72		0.1367	27		45	2025	75.00
3.0-3.9	20		0.1051	21		1	1	0.05
4.0-4.9	16		0.0807	16		0	0	0.00
5.0-5.9	7		0.0621	12		5	25	2.08
6.0-6.9	11		0.0477	10		1	1	0.10
7.0-7.9	6		0.0367	7		1	1	0.14
8.0-8.9	5		0.0282	6		1	1	0.17
9.0-9.9	3		0.0217	4				
10.0-10.9	2		0.0167	3				
11.0-11.9	1	9	0.0128	3	15	6	36	2.40
12.0-12.9	1		0.0099	2				
13.0-13.9	0		0.0075	2				
14.0-14.9	2		0.0059	1				
	N=200							Σ=127.39

$X^2_{(8,0.01)} = 20.09$

$127.39 > 20.09$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-14

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	3	0.0981	20	17	289	14.45
1.0-1.9	51	0.1853	37	14	196	5.30
2.0-2.9	72	0.1845	39	33	1089	27.92
3.0-3.9	20	0.1534	31	11	121	3.90
4.0-4.9	16	0.1170	23	7	49	2.13
5.0-5.9	7	0.0847	17	10	100	5.88
6.0-6.9	11	0.0590	12	1	1	0.08
7.0-7.9	6	0.0409	8	2	4	0.50
8.0-8.9	5	0.0266	5	0	0	0.00
9.0-9.9	3	0.0179	4			
10.0-10.9	2	0.0122	2			
11.0-11.9	1 } 9	0.0072	1 } 9	0	0	0.00
12.0-12.9	1	0.0046	1			
13.0-13.9	0	0.0036	1			
14.0-14.9	2	0.0014	0			
	N=200					Σ =60.16

$X^2_{(8,0.01)} = 20.09$

60.16 > 20.09 $\therefore$ Erlang does not fit at the 1% significance level.

TABLE D-15

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: PASSING LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	15	0.3611	72	57	3249	45.13
1.0-1.9	87	0.2307	46	41	1681	36.54
2.0-2.9	55	0.1474	29	26	676	23.31
3.0-3.9	26	0.0942	19	7	49	2.58
4.0-4.9	10	0.0601	12	2	4	0.33
5.0-5.9	4	0.0385	8	4	16	2.00
6.0-6.9	0	0.0245	5	5	25	5.00
7.0-7.9	2 } 3	0.0158	3 } 5	2	4	0.80
8.0-8.9	1	0.0100	2			
	N=200					Σ =115.69

$X^2_{(6,0.01)} = 16.81$

115.69 > 16.81 ∴ Poisson does not fit at the 1% significance level.

TABLE D-16

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
PASSING LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	15	0.2261	45	30	900	20.00
1.0-1.9	87	0.3088	62	25	625	10.08
2.0-2.9	55	0.2143	43	12	144	3.35
3.0-3.9	26	0.1238	25	1	1	0.04
4.0-4.9	10	0.0651	13	3	9	0.69
5.0-5.9	4	0.0326	7	3	9	1.29
6.0-6.9	0	0.0155	3 }			
7.0-7.9	2 } 3	0.0073	2 } 6	3	9	1.50
8.0-8.9	1 }	0.0056	1 }			
	N=200					Σ =36.95

$X^2_{(5,0.01)} = 15.09$

36.95 > 15.09 ∴ Erlang does not fit at the 1% significance level.

TABLE D-17

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.0870	15	14	196	13.07
1.0-1.9	13	0.0794	14	1	1	0.07
2.0-2.9	15	0.0725	13	2	4	0.31
3.0-3.9	21	0.0662	11	10	100	9.09
4.0-4.9	8	0.0605	10	2	4	0.40
5.0-5.9	13	0.0551	9	4	16	1.78
6.0-6.9	10	0.0504	9	1	1	0.11
7.0-7.9	6	0.0460	8	2	4	0.50
8.0-8.9	12	0.0420	7	5	25	3.57
9.0-9.9	6	0.0384	7	1	1	0.14
10.0-10.9	7	0.0350	6	1	1	0.17
11.0-11.9	8	0.0320	6	2	4	0.67
12.0-12.9	4	0.0291	5	1	1	0.20
13.0-13.9	5	0.0267	5	0	0	0.00
14.0-14.9	7 }	0.0243	4 }	5	25	0.51
≥15	37 } 44	0.2554	45 } 49			
	N=173					Σ=30.59

$X^2_{(13,0.01)} = 27.69$

$30.59 > 27.69$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-18

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1 14	0.0147	3 10	4	16	1.60
1.0-1.9	13	0.0375	7			
2.0-2.9	15	0.0522	9	6	36	4.00
3.0-3.9	21	0.0611	11	10	100	9.09
4.0-4.9	8	0.0657	11	3	9	0.82
5.0-5.9	13	0.0669	12	1	1	0.36
6.0-6.9	10	0.0659	11	1	1	0.09
7.0-7.9	6	0.0633	11	5	25	2.27
8.0-8.9	12	0.0599	10	2	4	0.40
9.0-9.9	6	0.0560	10	4	16	1.60
10.0-10.9	7	0.0512	9	2	4	0.44
11.0-11.9	8	0.0471	8	0	0	0.00
12.0-12.9	4	0.0424	7	3	9	1.29
13.0-13.9	5	0.0386	7	2	4	0.57
14.0-14.9	7	0.0343	6	1	1	0.17
≥15	37	0.2432	42	5	25	0.60
	N=173					Σ =23.30

$X^2_{(13,0.01)} = 27.69$

$23.30 < 27.69$ ∴ Erlang fits at the 1% significance level.

TABLE D-19

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)		PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)		(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	3		0.2212	44		41	1681	38.20
1.0-1.9	49		0.1660=	33		16	256	7.76
2.0-2.9	43		0.1312	26		17	289	11.12
3.0-3.9	28		0.1035	21		7	49	2.33
4.0-4.9	21		0.0818	16		5	25	1.56
5.0-5.9	11		0.0646	13		2	4	0.31
6.0-6.9	15		0.0510	10		5	25	2.50
7.0-7.9	12		0.0403	8		4	16	2.00
8.0-8.9	5		0.0318	6		1	1	0.17
9.0-9.9	4		0.0252	5		1	1	0.20
10.0-10.9	1		0.0198	4				
11.0-11.9	2		0.0157	3				
12.0-12.9	0		0.0125	3				
13.0-13.9	2	9	0.0097	2	16	7	49	3.06
14.0-14.9	2		0.0077	2				
15.0-15.9	1		0.0061	1				
16.0-16.9	1		0.0048	1				
	N=200							Σ=69.21

$X^2_{(9,0.01)} = 21.67$

$69.21 > 21.67$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-20

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: DRIVING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) K=2	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	3	0.0880	18	15	225	12.50
1.0-1.9	49	0.1556	31	18	324	10.45
2.0-2.9	43	0.1700	34	9	81	2.38
3.0-3.9	28	0.1492	30	2	4	0.13
4.0-4.9	21	0.1200	24	3	9	0.38
5.0-5.9	11	0.0915	18	7	49	2.72
6.0-6.9	15	0.0677	14	1	1	0.07
7.0-7.9	12	0.0486	10	2	4	0.40
8.0-8.9	5	0.0344	7	2	4	0.57
9.0-9.9	4	0.0241	5	1	1	0.20
10.0-10.9	1	0.0162	3			
11.0-11.9	2	0.0120	2			
12.0-12.9	0	0.0070	1			
13.0-13.9	2 } 9	0.0058	1 } 9	0	0	0.00
14.0-14.9	2	0.0034	1			
15.0-15.9	1	0.0022	1			
16.0-16.9	1	0.0016	0			
	N=200					Σ=29.80

$X^2_{(9,0.01)} = 21.67$

29.80 > 21.67 ∴ Erlang does not fit at the 1% significance level.

TABLE D-21

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
PASSING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	14	0.3923	78	64	4096	52.51
1.0-1.9	116	0.2383	48	68	4624	96.33
2.0-2.9	43	0.1449	30	13	169	5.63
3.0-3.9	14	0.0881	18	4	16	0.89
4.0-4.9	6	0.0535	11	5	25	2.27
5.0-5.9	2	0.0325	7	5	25	3.57
6.0-6.9	2	0.0197	4			
7.0-7.9	2	0.0121	2			
8.0-8.9	0	0.0073	2			
9.0-9.9	0 } 5	0.0044	1 } 10	5	25	2.50
10.0-10.9	0	0.0028	1			
11.0-11.9	0	0.0016	0			
12.0-12.9	1	0.0010	0			
	N=200					Σ=163.70

$X^2_{(5,0.01)} = 15.09$

$163.70 > 15.09$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-22

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: PASSING LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	14	0.2627	53	39	1521	28.70
1.0-1.9	116	0.3292	66	50	2500	37.88
2.0-2.9	43	0.2071	41	2	4	0.10
3.0-3.9	14	0.1083=	22	8	64	2.91
4.0-4.9	6	0.0514	10	4	16	1.60
5.0-5.9	2	0.0239	5			
6.0-6.9	2	0.0102	2			
7.0-7.9	2	0.0045	1			
8.0-8.9	0 } 7	0.0017	0 } 8	1	1	0.13
9.0-9.9	0	0.0005	0			
10.0-10.9	0	0.0003	0			
11.0-11.9	0	0.0001	0			
12.0-12.9	1	0.00006	0			
	N=200					Σ=71.32

$X^2_{(4,0.01)} = 13.28$

$71.32 > 13.28$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-23

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.1680	34	33	1089	32.03
1.0-1.9	50	0.1331	27	23	529	19.59
2.0-2.9	35	0.1120	22	13	169	7.68
3.0-3.9	28	0.0943	19	9	81	4.26
4.0-4.9	11	0.0794	16	5	25	1.56
5.0-5.9	20	0.0669	13	7	49	3.77
6.0-6.9	7	0.0563	11	4	16	1.45
7.0-7.9	11	0.0474	10	1	1	0.10
8.0-8.9	5	0.0399	8	3	9	1.13
9.0-9.9	3	0.0336	7	4	16	2.29
10.0-10.9	4	0.0283	6	2	4	0.67
11.0-11.9	2	0.0239	5	3	9	1.80
12.0-12.9	4	0.0200	4			
13.0-13.9	2 } 9	0.0169	3 } 10	1	1	0.10
14.0-14.9	3	0.0142	3			
≥15	14	0.0758	15	1	1	0.07
	N=200					Σ=76.50

$X^2_{(12,0.01)} = 26.22$

76.50 > 26.22 ∴ Poisson does not fit at the 1% significance level.

TABLE D-24

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
DECELERATION LANE - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	1	0.0472	9	8	64	7.11
1.0-1.9	50	0.1044	21	29	841	40.05
2.0-2.9	35	0.1244	25	10	100	4.00
3.0-3.9	28	0.1238	25	3	9	0.36
4.0-4.9	11	0.1130	23	12	144	6.26
5.0-5.9	20	0.0984	20	0	0	0.00
6.0-6.9	7	0.0821	16	9	81	5.06
7.0-7.9	11	0.0673	13	2	4	0.31
8.0-8.9	5	0.0543	11	6	36	3.27
9.0-9.9	3	0.0426	9	6	36	4.00
10.0-10.9	4	0.0339	7	3	9	1.29
11.0-11.9	2	0.0260	5	3	9	1.80
12.0-12.9	4 }	0.0202	4 }			
13.0-13.9	2 } 9	0.0153	3 } 9	0	0	0.00
14.0-14.9	3 }	0.0120	2 }			
≥15	14	0.0351	7	7	49	7.00
	N=200					Σ =80.51

$X^2_{(12,0.01)} = 26.22$

80.51 > 26.22 ∴ Erlang does not fit at the 1% significance level.

TABLE D-25

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
COMBINED THROUGH LANES - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	47	0.2673	53	6	36	0.68
1.0-1.9	59	0.1958	39	20	400	10.26
2.0-2.9	29	0.1435	29	0	0	0.00
3.0-3.9	15	0.1052	21	6	36	1.71
4.0-4.9	13	0.0770	15	2	4	0.27
5.0-5.9	10	0.0565	11	1	1	0.09
6.0-6.9	5	0.0413	8	3	9	1.13
7.0-7.9	5	0.0303	6	1	1	0.17
8.0-8.9	9	0.0222	4			
9.0-9.9	3	0.0163	3			
10.0-10.9	2 } 17	0.0119	3 } 14	3	9	0.64
11.0-11.9	2	0.0087	2			
12.0-12.9	0	0.0065	1			
13.0-13.9	1	0.0046	1			
	N=200					Σ=14.95

$X^2_{(7,0.025)} = 16.01$

$14.95 < 16.01$ ∴ Poisson fits at the 2.5% significance level.

TABLE D-26

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: COMBINED THROUGH LANES - DIXON ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	47	0.1291	26	21	441	16.96
1.0-1.9	59	0.2242	45	14	196	4.36
2.0-2.9	29	0.2033	41	12	144	3.51
3.0-3.9	15	0.1535	31	16	256	8.26
4.0-4.9	13	0.1066	21	8	64	3.05
5.0-5.9	10	0.0697	14	4	16	1.14
6.0-6.9	5	0.0445	9	4	16	1.78
7.0-7.9	5	0.0279	6	1	1	0.17
8.0-8.9	9	0.0168	3			
9.0-9.9	3	0.0100	2			
10.0-10.9	2 } 17	0.0058	1 } 7	10	100	14.29
11.0-11.9	2	0.0035	1			
12.0-12.9	0	0.0024	0			
13.0-13.9	1	0.0008	0			
	N=200					$\Sigma=53.52$

$X^2_{(7,\ 0.01)} = 18.48$

$53.52 > 18.48$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-27

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
COMBINED THROUGH LANES - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	44	0.3693	74	30	900	12.16
1.0-1.9	75	0.2330	47	28	784	16.68
2.0-2.9	33	0.1469	29	4	16	0.55
3.0-3.9	18	0.0926	19	1	1	0.06
4.0-4.9	9	0.0582	12	3	9	0.75
5.0-5.9	9	0.0371	7	2	4	0.58
6.0-6.9	6	0.0233	5	1	1	0.20
7.0-7.9	1	0.0146	3			
8.0-8.9	2	0.0092	2			
9.0-9.9	2 } 6	0.0059	1 } 7	1	1	0.14
10.0-10.9	0	0.0036	1			
11.0-11.9	1	0.0023	0			
	N=200					Σ =31.12

$X^2_{(6,0.01)} = 16.81$

31.12 > 16.81 $\therefore$ Poisson does not fit at the 1% significance level.

TABLE D-28

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
COMBINED THROUGH LANES - ISLINGTON AVE.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	44	0.2356	47	3	9	0.19
1.0-1.9	75	0.3145	63	12	144	2.29
2.0-2.9	33	0.2130	43	10	100	2.33
3.0-3.9	18	0.1197	24	6	36	1.50
4.0-4.9	9	0.0617	12	3	9	0.75
5.0-5.9	9	0.0268	5	4	16	3.20
6.0-6.9	6	0.0168	3			
7.0-7.9	1	0.0069	1			
8.0-8.9	2 } 12	0.0031	1 } 5	7	49	9.80
9.0-9.9	2	0.0009	0			
10.0-10.9	0	0.0006	0			
11.0-11.9	1	0.0002	0			
	N=200					$\Sigma = 20.06$

$X^2_{(5,0.01)} = 18.48$

$20.06 > 18.48$ ∴ Erlang does not fit at the 1% significance level.

TABLE D-29

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF POISSON DISTRIBUTION TO OBSERVED DATA:
COMBINED THROUGH LANES - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	61	0.5093	102	41	1681	16.48
1.0-1.9	87	0.2500	50	27	729	14.58
2.0-2.9	33	0.1226	25	8	64	2.56
3.0-3.9	14	0.0601	12	2	4	0.33
4.0-4.9	3	0.0296	6	3	9	1.50
5.0-5.9	1	0.0144	3			
6.0-6.9	0 } 2	0.0072	1 } 5	3	9	1.80
7.0-7.9	0	0.0035	1			
8.0-8.9	1	0.0017	0			
	N=200					$\Sigma = 37.25$

$X^2_{(4,0.01)} = 13.28$

$37.25 > 13.28$ ∴ Poisson does not fit at the 1% significance level.

TABLE D-30

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT
OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA:
COMBINED THROUGH LANES - DUFFERIN ST.

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	61	0.4165	83	22	484	5.83
1.0-1.9	87	0.3603	72	15	225	3.13
2.0-2.9	33	0.1494	30	3	9	0.30
3.0-3.9	14	0.0517	10			
4.0-4.9	3	0.0156	3			
5.0-5.9	1	0.0046	1			
6.0-6.9	0 } 19	0.0014	0 } 14	5	25	1.78
7.0-7.9	0	0.0004	0			
8.0-8.9	1	0.00006	0			
=	N=200					Σ=11.04

$X^2_{(2,0.01)} = 9.21$

11.04 > 9.21 ∴ Erlang does not fit at the 1% significance level.

TABLE D-31

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF POISSON DISTRIBUTION TO OBSERVED DATA: COMBINED THROUGH LANES - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (POISSON)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	62	0.5195	104	42	1764	16.96
1.0-1.9	95	0.2497	50	45	2025	40.50
2.0-2.9	26	0.1199	24	2	4	0.16
3.0-3.9	9	0.0576	12	3	9	0.75
4.0-4.9	4	0.0277	6	2	4	0.67
5.0-5.9	2	0.0133	3			
6.0-6.9	1 } 4	0.0064	1 } 5	1	1	0.20
7.0-7.9	1	0.0031	1			
	N=200					Σ =59.24

$X^2_{(4,0.01)} = 13.28$

$59.24 > 13.28$ ·)· Poisson does not fit at the 1% significance level.

TABLE D-32

CHI-SQUARE TEST TO CHECK THE GOODNESS OF FIT OF ERLANG (K=2) DISTRIBUTION TO OBSERVED DATA: COMBINED THROUGH LANES - AVENUE ROAD

CLASS INTERVAL (SEC)	OBSERVED FREQUENCY (fo)	PROBABILITY (ERLANG) (K=2)	THEORETICAL FREQUENCY (fT)	(fo-fT)	$(fo-fT)^2$	$\frac{(fo-fT)^2}{fT}$
0.0-0.9	62	0.4308	86	14	196	2.28
1.0-1.9	95	0.3596	72	23	529	7.35
2.0-2.9	26	0.1432	29	3	9	0.31
3.0-3.9	9	0.0472	9			
4.0-4.9	4	0.0134	3			
5.0-5.9	2 } 17	0.0038	1 } 13	4	16	1.23
6.0-6.9	1	0.0015	0			
7.0-7.9	1	0.0004	0			
=	N=200					Σ=11.17

$X^2_{(2,0.01)} = 9.21$

11.17 > 9.21 ∴ Erlang does not fit at the 1% significance level.

REFERENCES

1. Bennett, C.A., and Franklin, N.L., Statistical Analysis in Chemistry and the Chemical Industry; John Wiley and Sons, 1954.

2. Greenshields, B.D., and Weida, F.M.; Statistics with applications to Highway Traffic Analysis; the Eno Foundation for Highway Traffic Control, 1952.

3. Gerlough, D.L.; "The use of the Poisson Distribution in Highway Traffic", in Gerlough, D.L., and Schuhl, A., "Poisson and Traffic"; the Eno Foundation for Highway Traffic Control, 1955.

4. Haight, F.A., Whisler, B.F., and Mosher, W.W.Jr.; A new statistical method for describing the distribution of cars on a road; Institute of Transportation and Traffic Engineering, University of California, Los Angeles.

5. Jensen, A.; "Traffic theory as an aid in the planning and operation of the Road Grid". Ingenitren No.2, vol. 1, 1957. (Danish Engineering Periodical).

6. Kendall, D.G.; Some problems in the Theory of Queues; Royal Statistical Journal, Series 13, Vol. 13, No.2, 1951.

7. Matson, T.M., Smith, W.S., and Hurd, F.W.; Traffic Engineering; McGraw-Hill, 1955.

8. Morse, P.M.; Queues, Inventories and Maintenance; Operation Research Society of America, publication No.1; John Wiley and Sons, 1952.

9. Pearson, K.; Tables for Statisticians and Biometricians; Biometric Laboratory, University College, London, part I, third edition, 1951.

10. Smith, J.G., and Duncan, A.J.; Sampling Statistics and applications; McGraw-Hill, 1943.

11. Vardon, J.L.; "Some factors affecting merging traffic on the outer Ramp of Highway Interchanges"; A Master's degree thesis, Queen's University, July 1959.

12. Williams, K.M.; "Vehicle operating characteristics on outer loop deceleration lanes of interchanges"; A Master's degree thesis, University of Toronto, October 1960.

13. Wyse, J.M., "A study of some characteristics of Traffic Flow on Highway 401, an Urban Freeway"; Unpublished Technical Report of the Ontario Department of Highways, 1960.